0/1 Knapsack Problem

Example

5.6. That is, $n = 4$, $W = 16$, and we have the following:

i	p_i	w_i	$\frac{p_i}{w_i}$
1	\$40	2	\$20
2	\$30	5	\$6
3	\$50	10	\$5
4	\$10	5	\$2

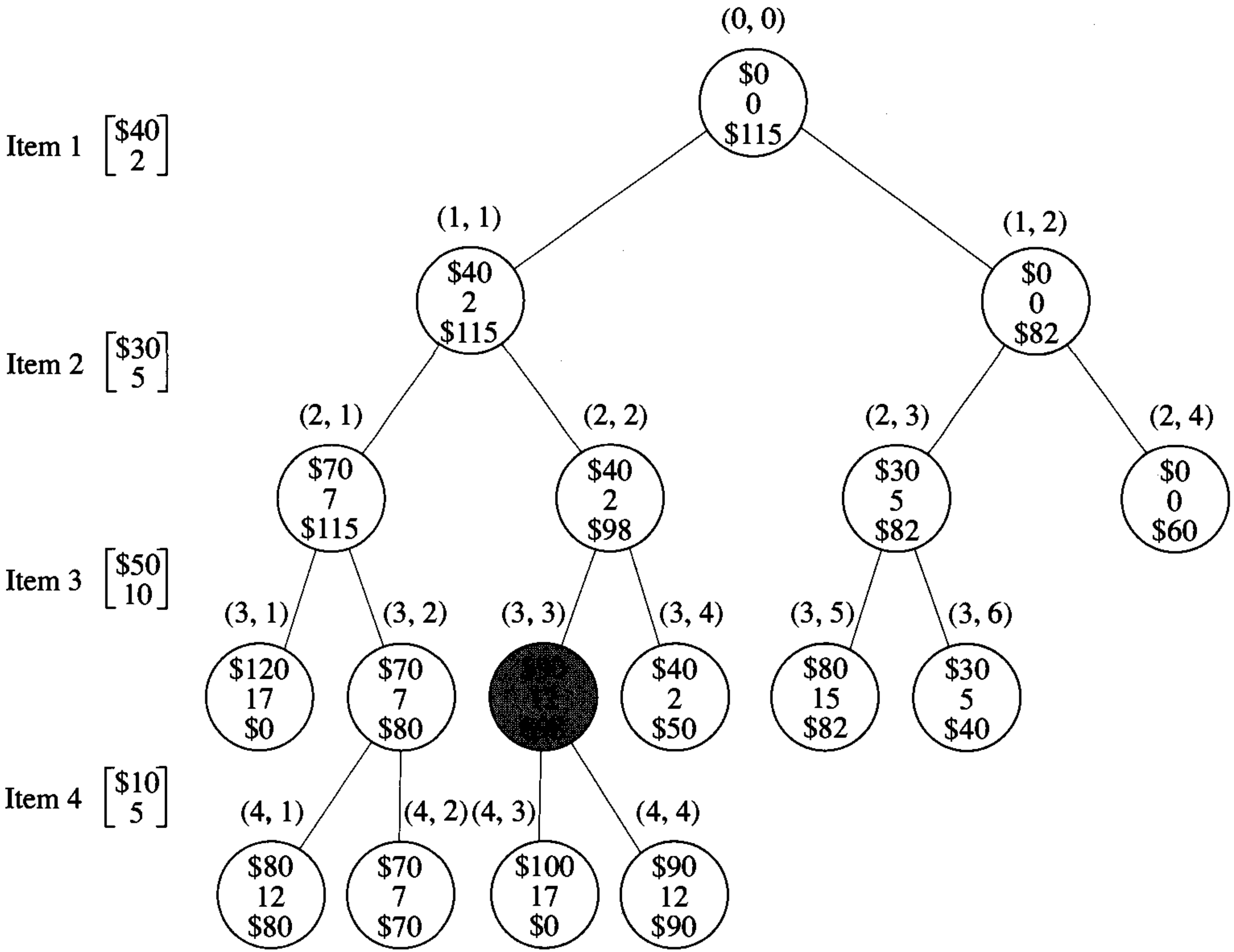
$$totweight = weight + \sum_{j=i+1}^{k-1} w_j$$

and

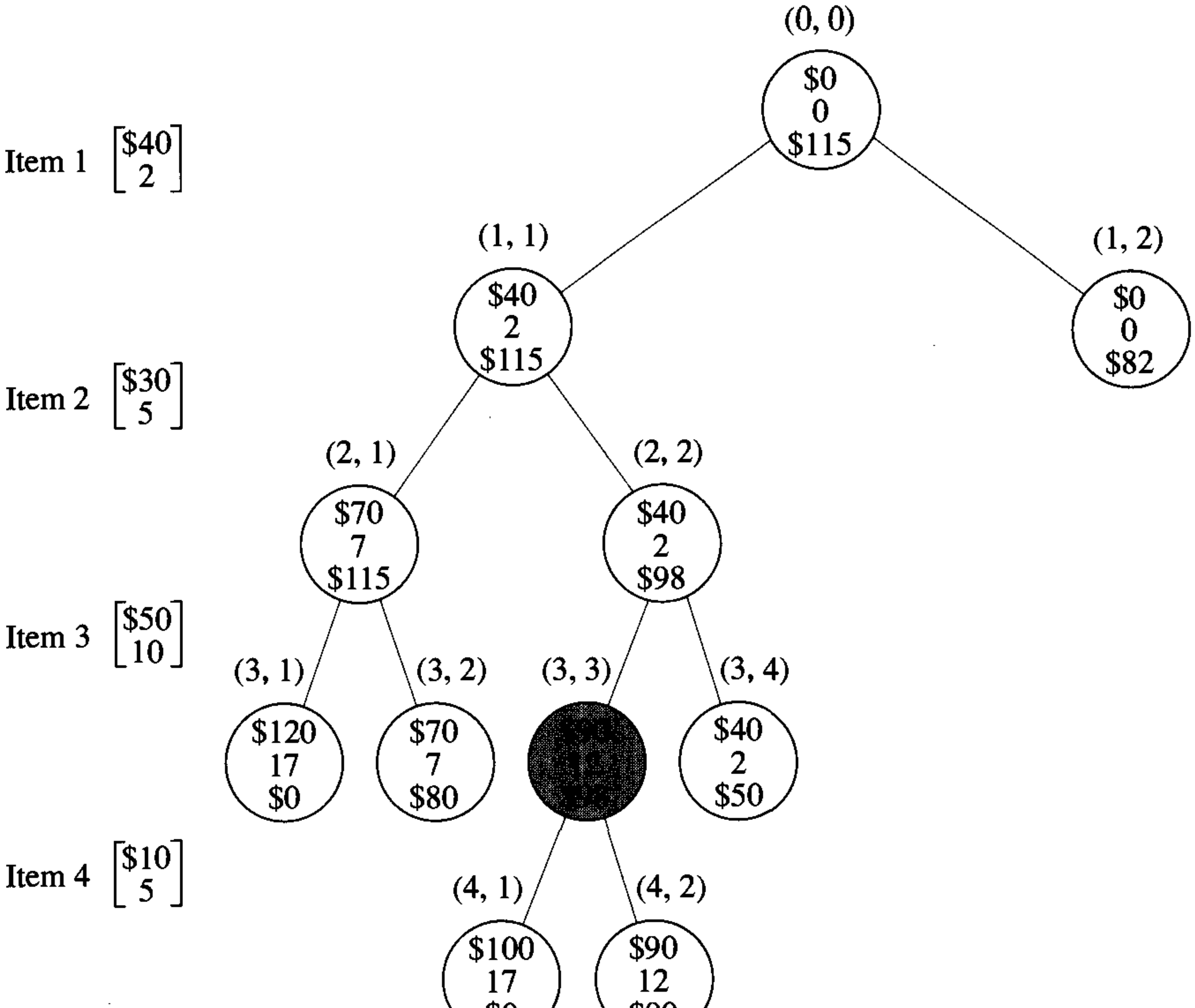
$$bound = \left(profit + \sum_{j=i+1}^{k-1} p_j\right) + (W - totweight) \times \frac{p_k}{w_k}.$$

Non Promising node

$$weight \geq W.$$



Item 1 [$40 2]
Item 2 [$30 5]
Item 3 [$50 10]
Item 4 [$10 5]
(0, 0) $0 0 $115
(1, 1) $40 2 $115
(1, 2) $0 0 $82
(2, 1) $70 7 $115
(2, 2) $40 2 $98
(2, 3) $30 5 $82
(2, 4) $0 0 $60
(3, 1) $120 17 $0
(3, 2) $70 7 $80
(3, 3)
(3, 4) $40 2 $50
(3, 5) $80 15 $82
(3, 6) $30 5 $40
(4, 1) $80 12 $80
(4, 2) $70 7 $70
(4, 3) $100 17 $0
(4, 4) $90 12 $90


Item 1 [$40 2]
Item 2 [$30 5]
Item 3 [$50 10]
Item 4 [$10 5]
(0, 0)
$0
0
$115
(1, 1)
$40
2
$115
(1, 2)
$0
0
$82
(2, 1)
$70
7
$115
(2, 2)
$40
2
$98
(3, 1)
$120
17
$0
(3, 2)
$70
7
$80
(3, 3)
(3, 4)
$40
2
$50
(4, 1)
$100
17
(4, 2)
$90
12

Node(0,0)

1. Set *maxprofit* to \$0.
2. Visit node (0, 0) (the root).
   (a) Compute its profit and weight.

$$profit = \$0$$
$$weight = 0$$

   (b) Compute its bound. Because $2 + 5 + 10 = 17$, and $17 > 16$, the value of W, the third item would bring the sum of the weights above W. Therefore, $k = 3$, and we have

$$totweight = weight + \sum_{j=0+1}^{3-1} w_j = 0 + 2 + 5 = 7$$

$$bound = profit + \sum_{j=0+1}^{3-1} p_j + (W - totweight) \times \frac{p_3}{w_3}$$
$$= \$0 + \$40 + \$30 + (16 - 7) \times \frac{\$50}{10} = \$115.$$

   (c) Determine that it is promising because its weight 0 is less than 16, the value of W, and its bound \$115 is greater than \$0, the value of *maxprofit*.

3. Visit node (1, 1).
   (a) Compute its profit and weight.

$$\begin{aligned} \textit{profit} &= \$0 + \$40 = \$40 \\ \textit{weight} &= 0 + 2 = 2 \end{aligned}$$

   (b) Because its weight 2 is less than or equal to 16, the value of *W*, and its profit \$40 is greater than \$0, the value of *maxprofit*, set *maxprofit* to \$40.
   (c) Compute its bound. Because $2 + 5 + 10 = 17$, and $17 > 16$, the value of *W*, the third item would bring the sum of the weights above *W*. Therefore, $k = 3$, and we have

$$\textit{totweight} = \textit{weight} + \sum_{j=1+1}^{3-1} w_j = 2 + 5 = 7$$

$$\begin{aligned} \textit{bound} &= \textit{profit} + \sum_{j=1+1}^{3-1} p_j + (W - \textit{totweight}) \times \frac{p_3}{w_3} \\ &= \$40 + \$30 + (16 - 7) \times \frac{\$50}{10} = \$115. \end{aligned}$$

   (d) Determine that it is promising because its weight 2 is less than 16, the value of *W*, and its bound \$115 is greater than \$0, the value of *maxprofit*.

2. Visit node (1, 1).
   (a) Compute its profit and weight to be $40 and 2.

   (b) Because its weight 2 is less than or equal to 16, the value of *W*, and its profit $40 is greater than $0, the value of *maxprofit*, set *maxprofit* to $40.
   (c) Compute its bound to be $115.

3. Visit node (1, 2).
   (a) Compute its profit and weight to be \$0 and 0.
   (b) Compute its bound to be \$82.

4. Determine promising, unexpanded node with the greatest bound.
   (a) Because node (1, 1) has a bound of \$115 and node (1, 2) has a bound of \$82, node (1, 1) is the promising, unexpanded node with the greatest bound. We visit its children next.

5. Visit node (2, 1).
   (a) Compute its profit and weight to be \$70 and 7.
   (b) Because its weight 7 is less than or equal to 16, the value of W, and its profit \$70 is greater than \$40, the value of *maxprofit*, set *maxprofit* to \$70.
   (c) Compute its bound to be \$115.

6. Visit node (2, 2).
   (a) Compute its profit and weight to be \$40 and 2.
   (b) Compute its bound to be \$98.

7. Determine promising, unexpanded node with the greatest bound.
   (a) That node is node (2, 1). We visit its children next.

8. Visit node (3, 1).
   (a) Compute its profit and weight to be \$120 and 17.
   (b) Determine that it is nonpromising because its weight 17 is greater than or equal to 16, the value of *W*. We make it nonpromising by setting its bound to \$0.

9. Visit node (3, 2).
   (a) Compute its profit and weight to be \$70 and 7.
   (b) Compute its bound to be \$80.

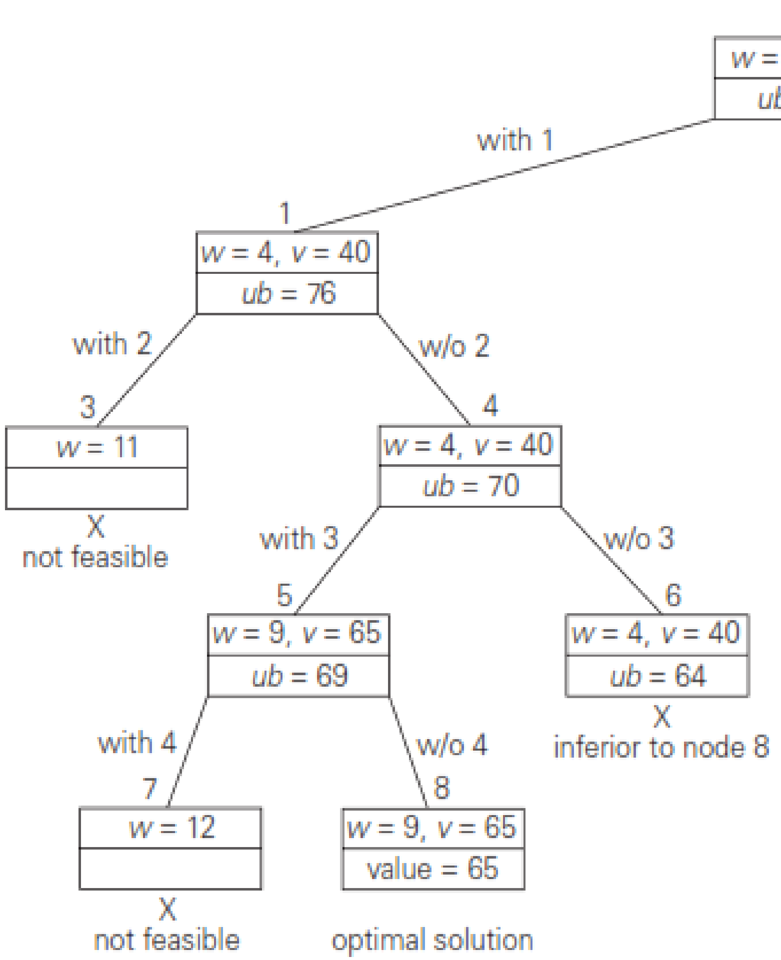
10. Determine promising, unexpanded node with the greatest bound.
    (a) That node is node (2, 2). We visit its children next.

11. Visit node (3, 3).
    (a) Compute its profit and weight to be \$90 and 12.
    (b) Because its weight 12 is less than or equal to 16, the value of W, and its profit \$90 is greater than \$70, the value of *maxprofit*, set *maxprofit* to \$90.
    (c) At this point, nodes (1, 2) and (3, 2) become nonpromising because their bounds, \$82 and \$80 respectively, are less than or equal to \$90, the new value of *maxprofit*.
    (d) Compute its bound to be \$98.

12. Visit node (3, 4).
    (a) Compute its profit and weight to be \$40 and 2.
    (b) Compute its bound to be \$50.
    (c) Determine that it is nonpromising because its bound \$50 is less than or equal to \$90, the value of *maxprofit*.
13. Determine promising, unexpanded node with the greatest bound.
    (a) The only unexpanded, promising node is node (3, 3). We visit its children next.
14. Visit node (4, 1).
    (a) Compute its profit and weight to be \$100 and 17.
    (b) Determine that it is nonpromising because its weight 17 is greater than or equal to 16, the value of W. We set its bound to \$0.
15. Visit node (4, 2).
    (a) Compute its profit and weight to be \$90 and 12.
    (b) Compute its bound to be \$90.
    (c) Determine that it is nonpromising because its bound \$90 is less than or equal to \$90, the value of *maxprofit*. Leaves in the state space tree are automatically nonpromising because their bounds cannot exceed *maxprofit*.

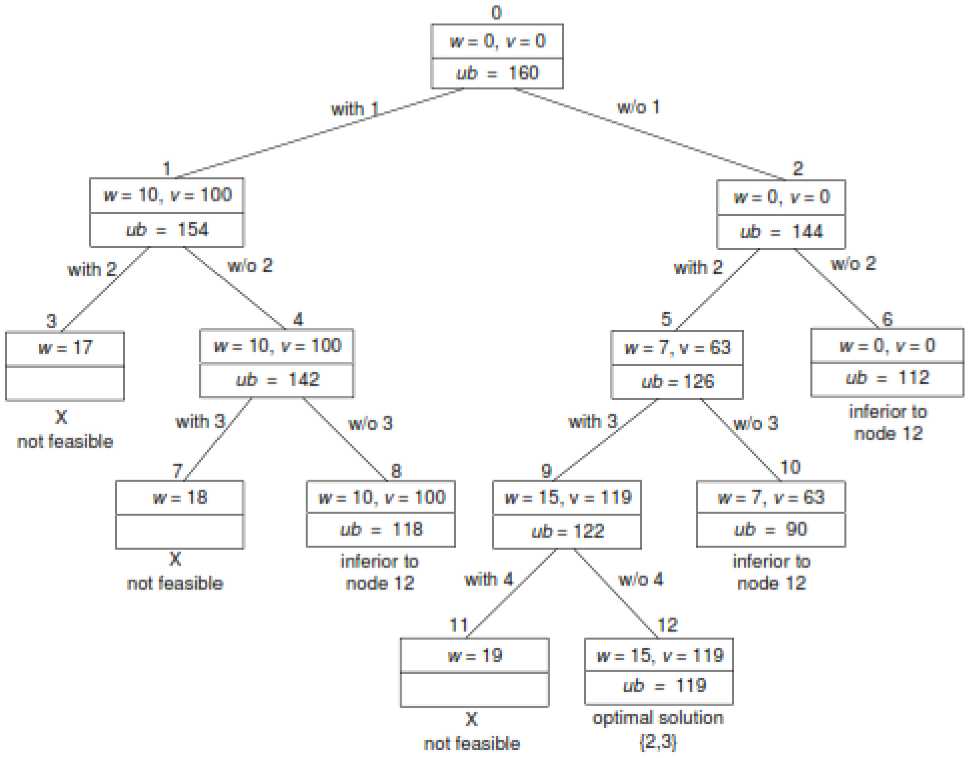
item	weight	value	$\frac{\text{value}}{\text{weight}}$
1	4	$40	10
2	7	$42	6
3	5	$25	5
4	3	$12	4

The knapsack's capacity *W* is 10.


0
w = 0, v = 0
ub = 100
with 1
w/o 1
1
w = 4, v = 40
ub = 76
2
w = 0, v = 0
ub = 60
X
inferior to node 8
with 2
w/o 2
3
w = 11
X
not feasible
4
w = 4, v = 40
ub = 70
with 3
w/o 3
5
w = 9, v = 65
ub = 69
6
w = 4, v = 40
ub = 64
X
inferior to node 8
with 4
w/o 4
7
w = 12
X
not feasible
8
w = 9, v = 65
value = 65
optimal solution

item	**weight**	**value**
1	10	\$100
2	7	\$63
3	8	\$56
4	4	\$12

$W = 16$



The found optimal solution is {item 2, item 3} of value $119.