Faculty Development Program

Design and Analysis of Algorithms

Brute Force and Exhaustive Search

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SSN College of Engineering

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Postpone it as long as possible!

Problems

- Selection sort
- Insertion sort
- Closest pair of points
- Convex Hull
- Traveling Salesman Problem
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- Assignment

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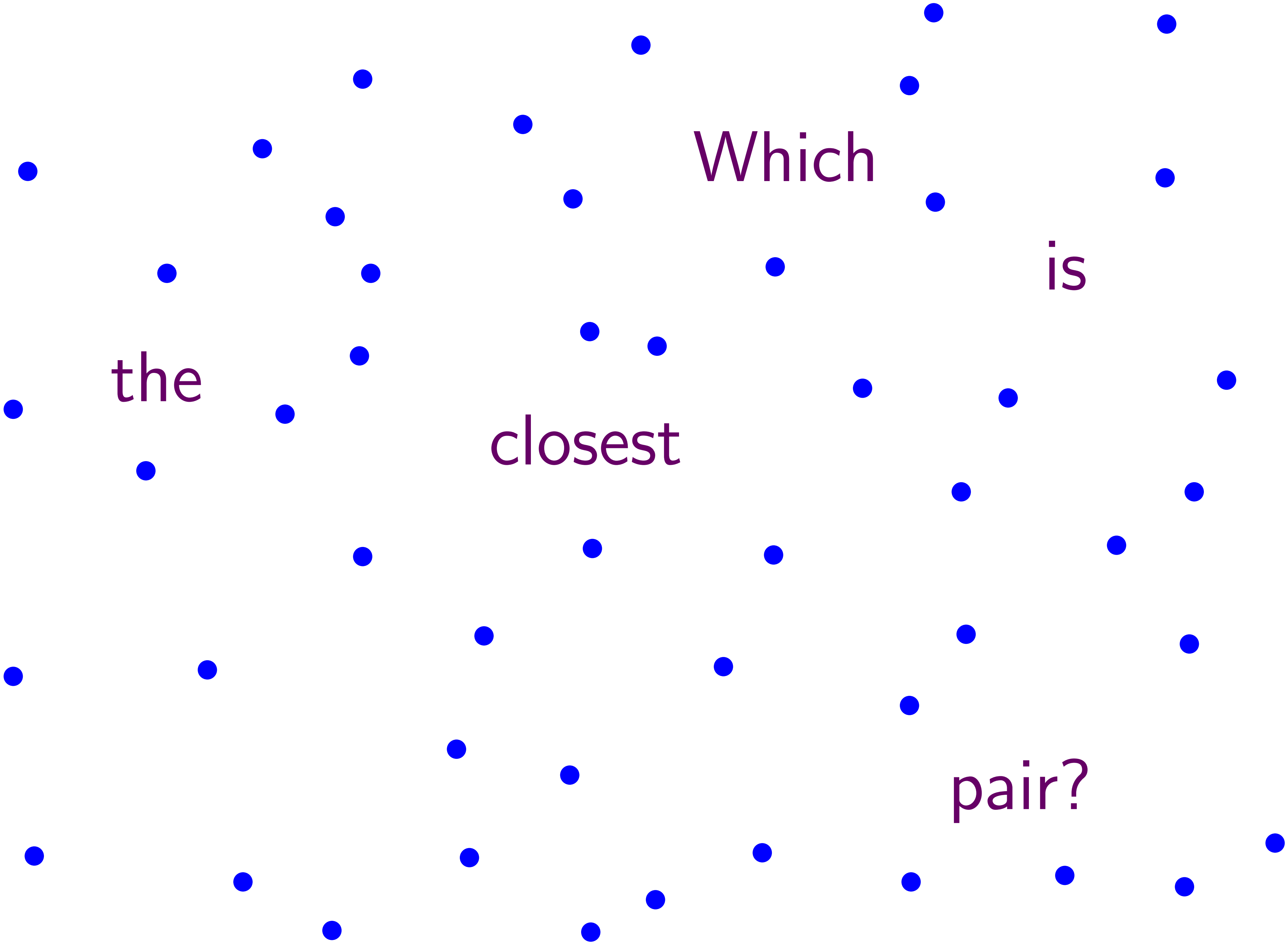

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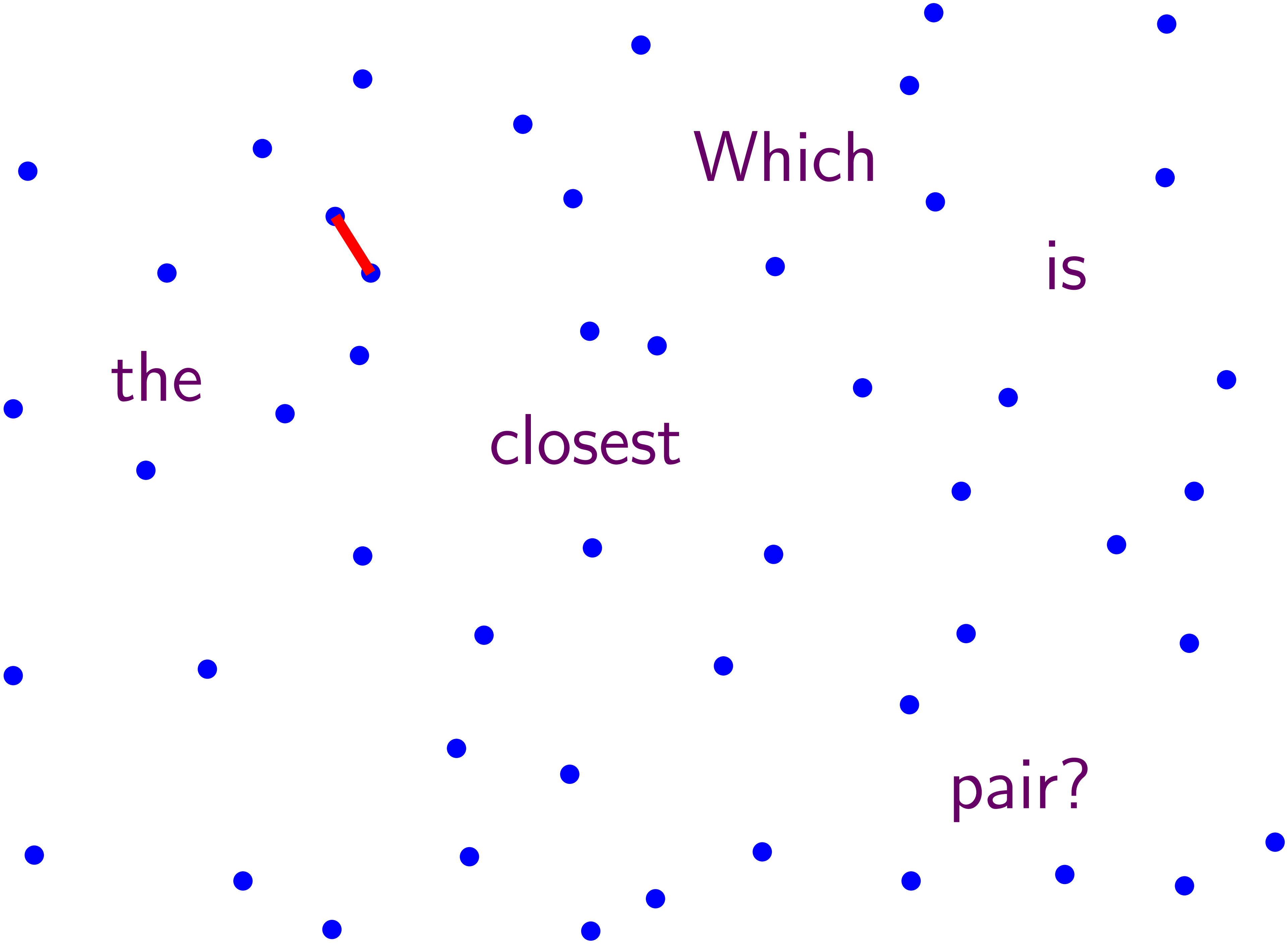

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Input: A list P of n points $p_1(x_1, y_1), \ldots p_n(x_n, y_n)$

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Definition: Distance between p_i and p_j

$$d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

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$(n-(i+1)+1)2$

$(n - i)2$

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5. return d

$$= 2((n-1) + \ldots + 1) = 2\frac{(n-1)n}{2} = (n-1)n = n^2 - n = O(n^2)$$

Convex Set

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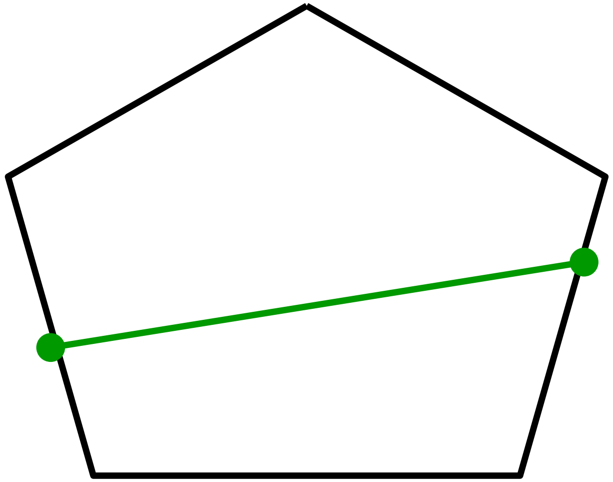

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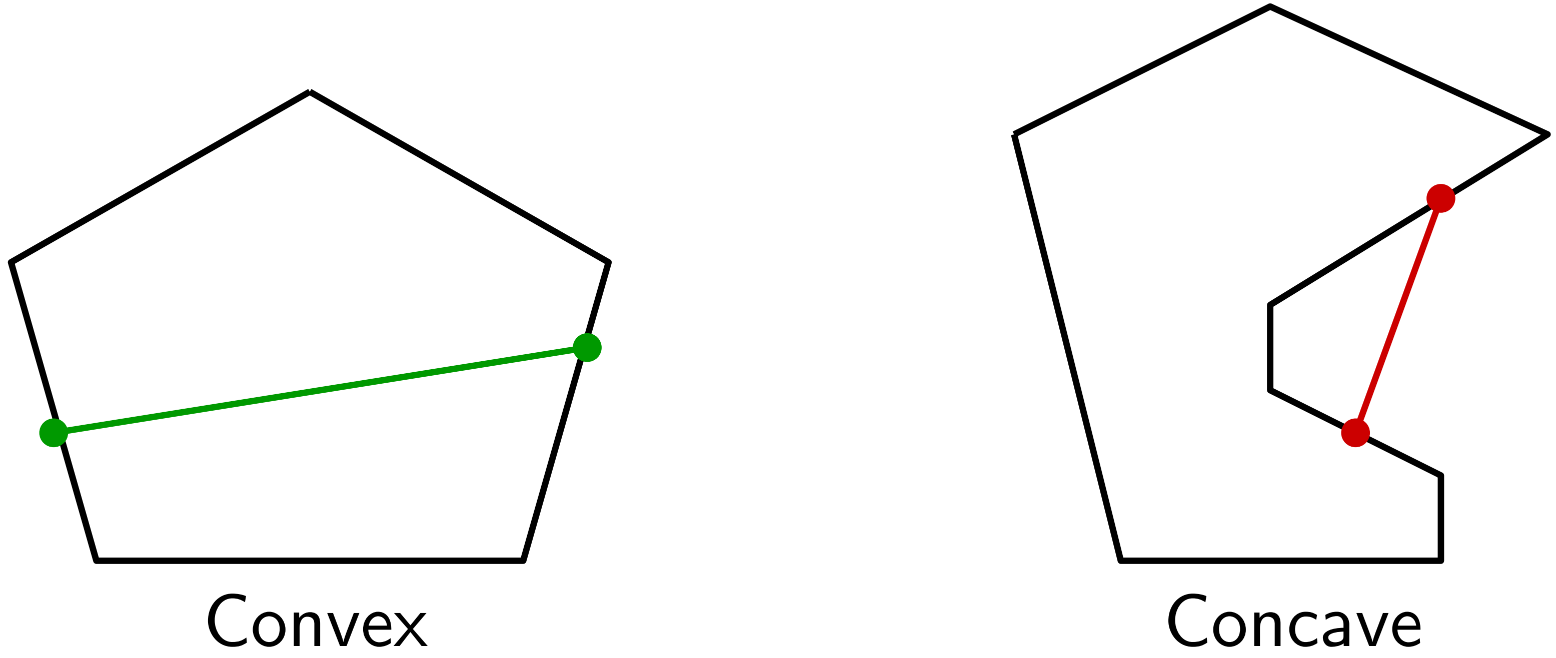

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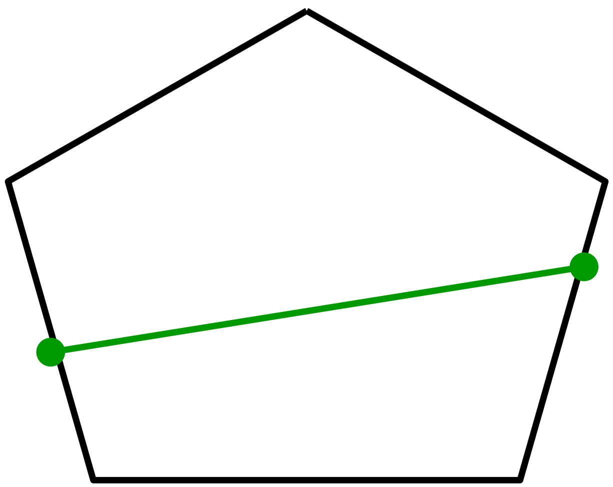

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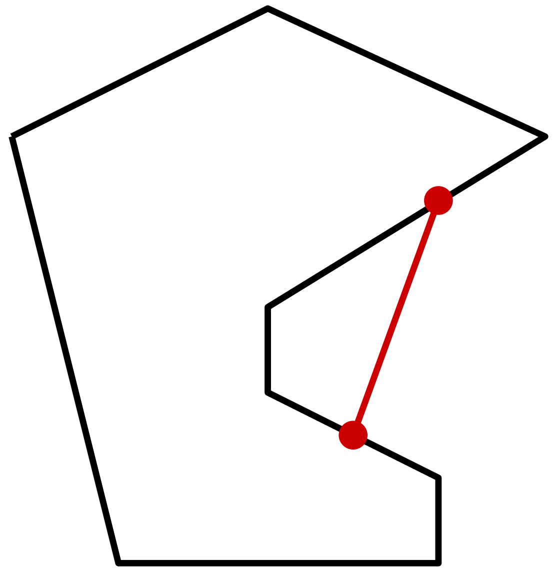

Concave

- Straight line
- Triangle
- Rectangle
- Any convex polygon

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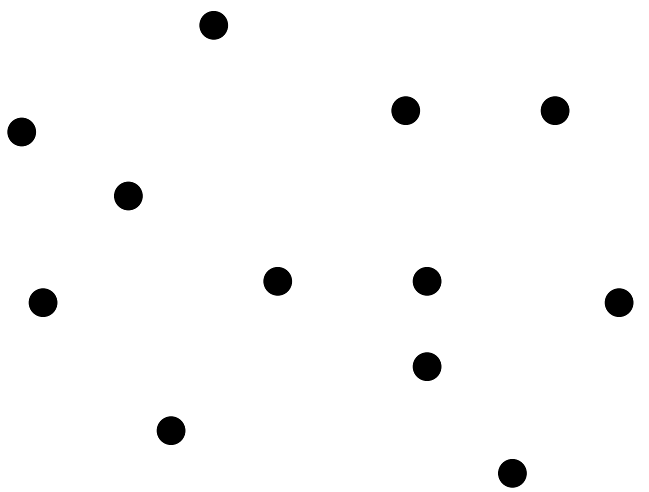

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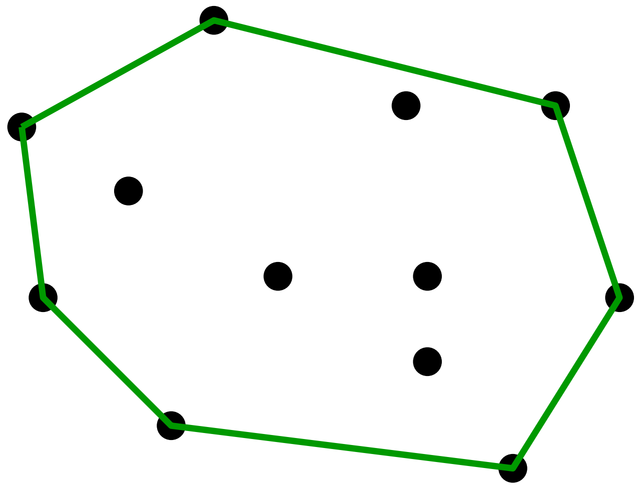

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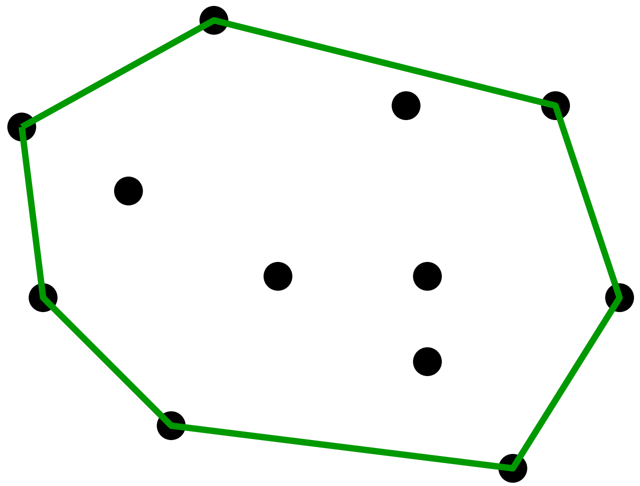
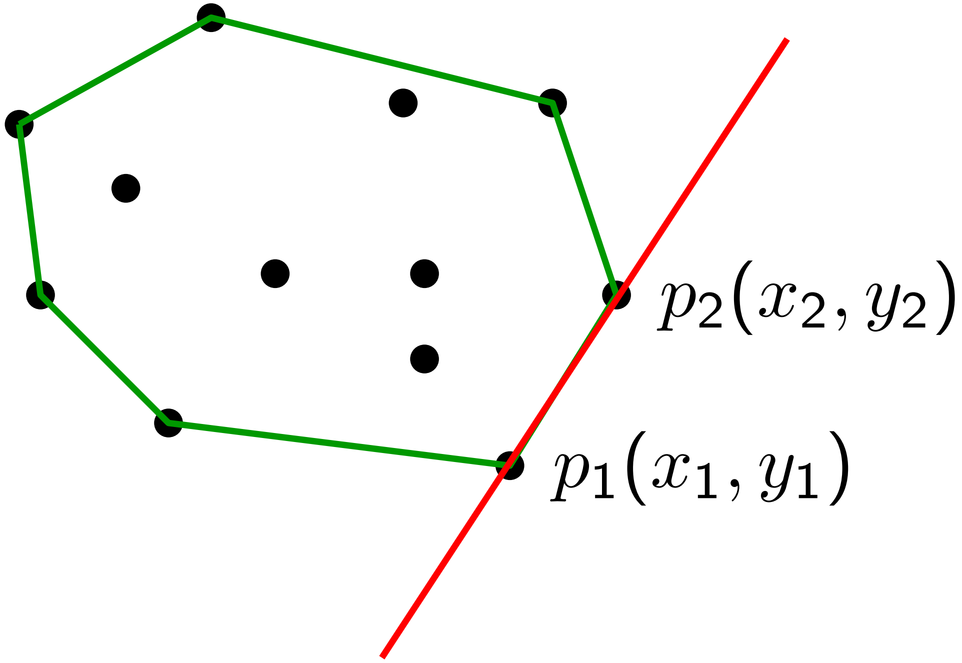


Convex hull(formal): The convex hull of a set S of points is the smallest convex set containing S.

Algorithm

A line segment connecting two points p_1 and p_2 of a set of n points is a part of the convex hull's boundary if and only if all the other points of the set lie on the same side of the straight line through these two points.



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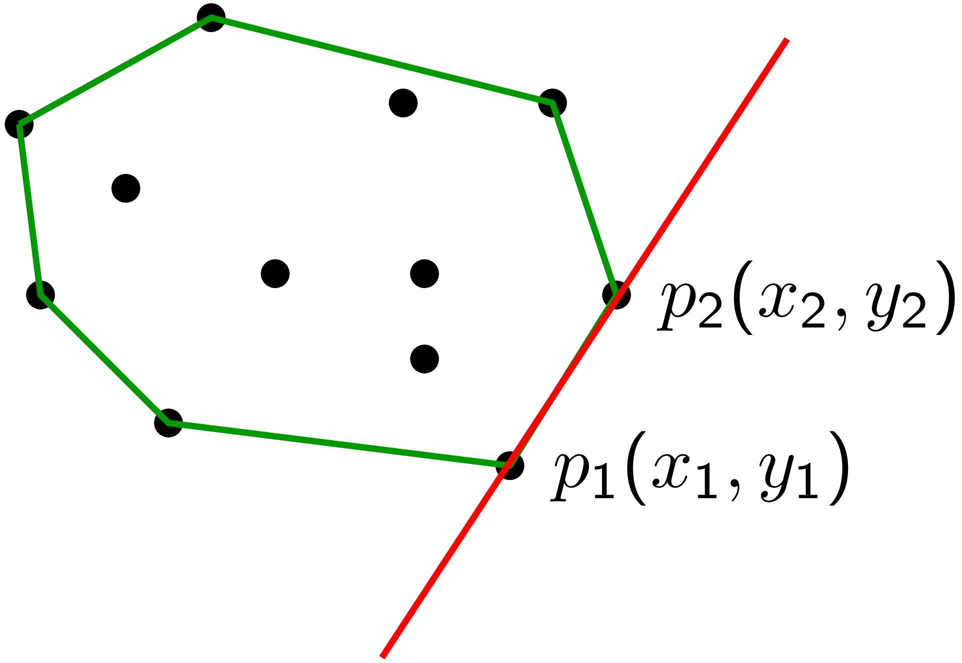

Striaght line with two points $(x_1, y_1), (x_2, y_2)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$ax + by = c$$

$$a = y_2 - y_1$$

$$b = x_1 - x_2$$

$$c = x_1 y_2 - y_1 x_2$$

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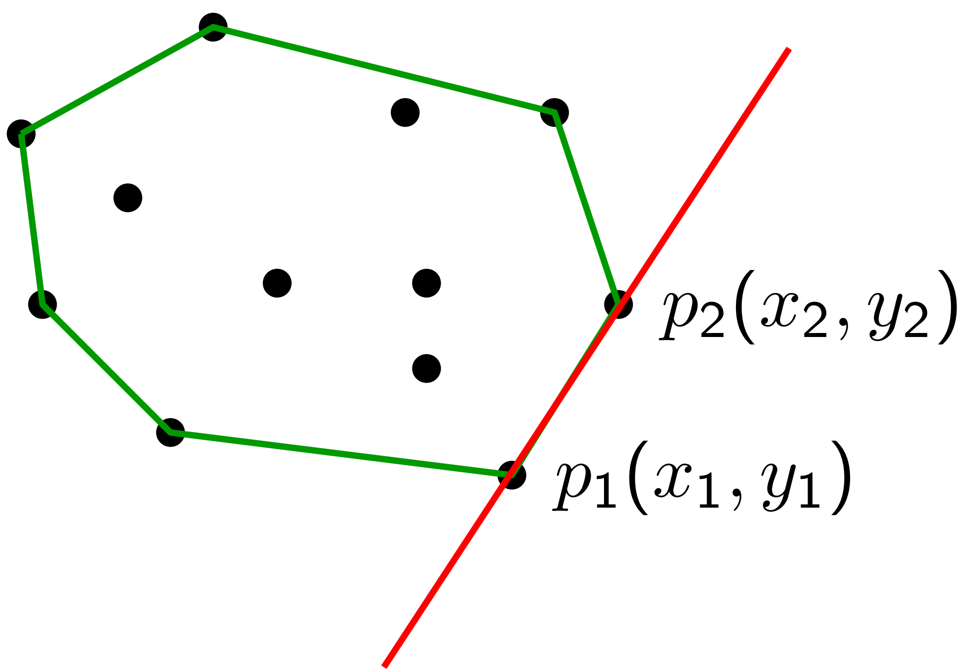

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Do certain points lie on the same side of the line?
$\equiv$ Does $ax + by - c$ have the same sign for each of these points?

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- Running time $= O(n^3)$

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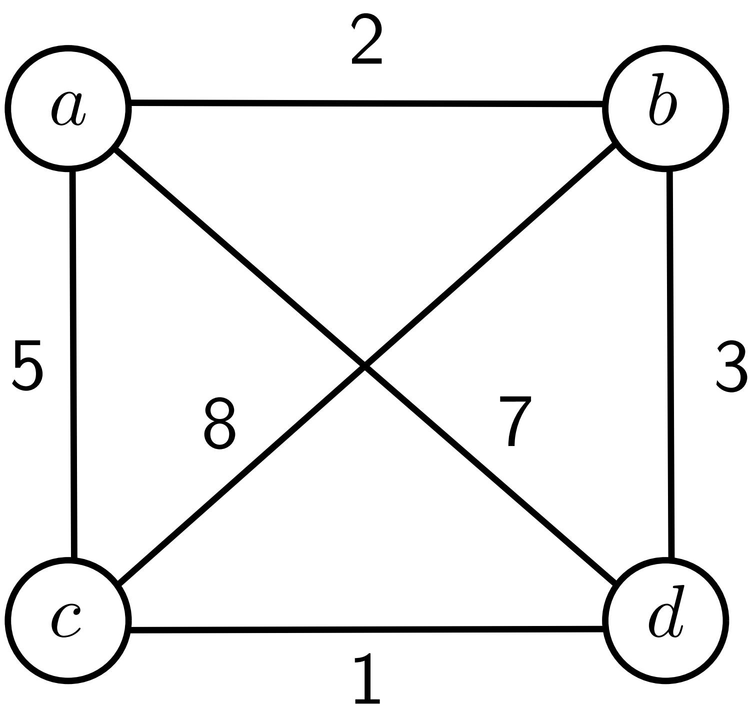

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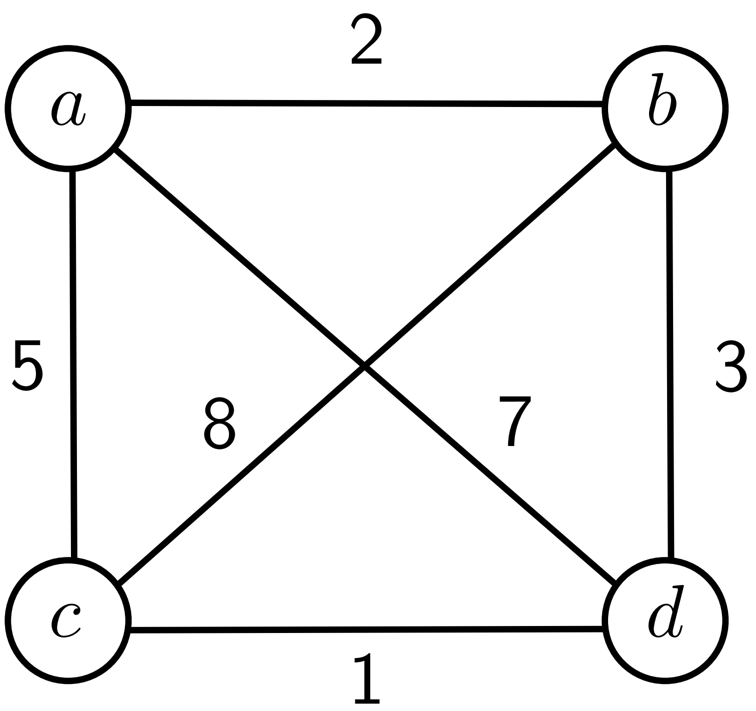

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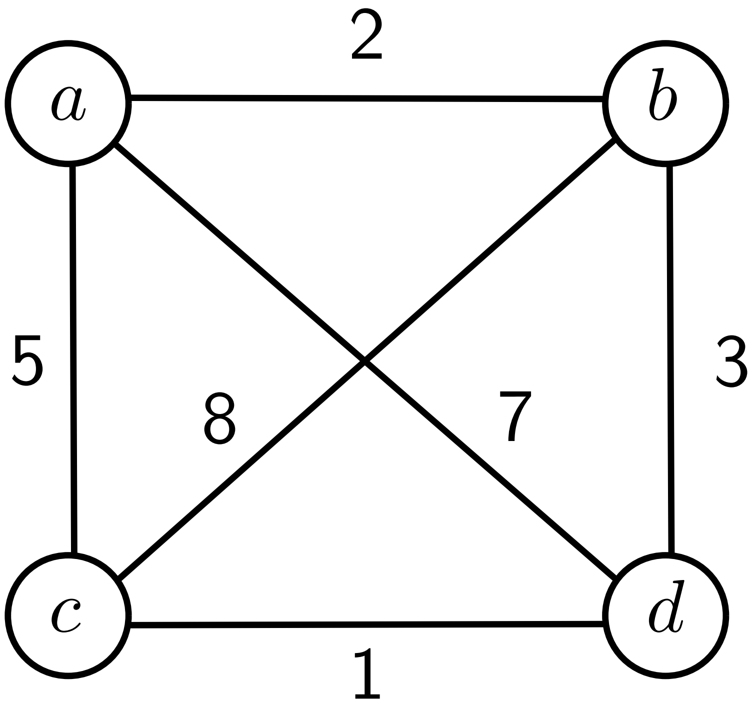

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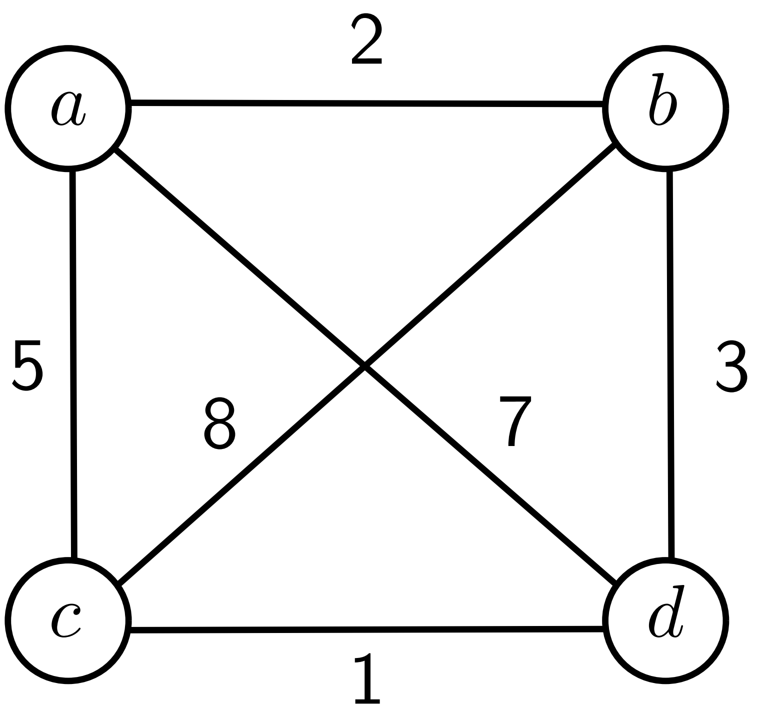

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A Hamiltonian circuit is a cycle that passes through all the vertices of the graph exactly once.

TSP: Find the shortest Hamiltonian circuit of the graph.

Hamiltonian Circuit

- A sequence of $n+1$ adjacent vertices $v_{i_0}, v_{i_1}, \ldots, v_{i_n-1}, v_{i_0}$.
- First vertex and the last vertex are the same.
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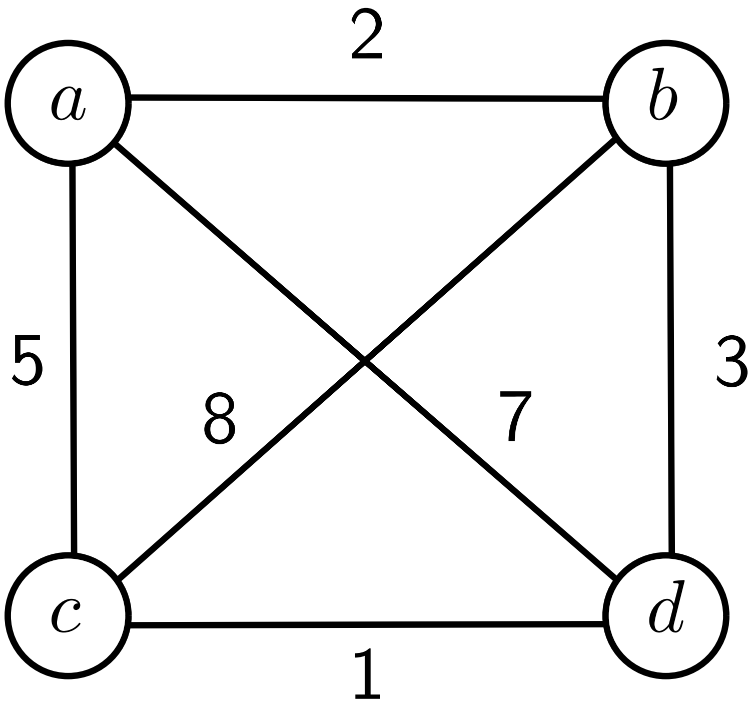

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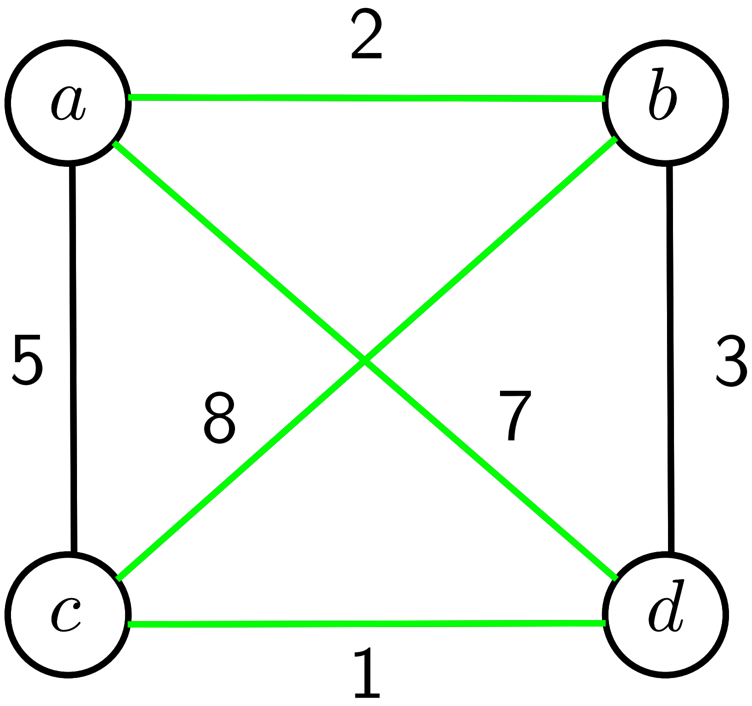
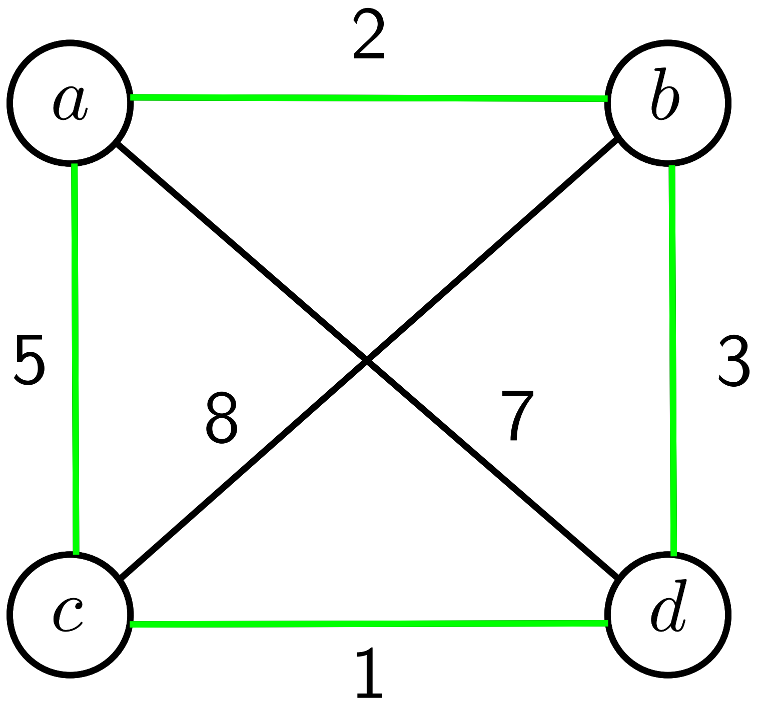


$a \rightarrow b \rightarrow c \rightarrow d \rightarrow a \qquad 2 + 8 + 1 + 7 = 18$

Example



$$a \to b \to c \to d \to a \qquad 2 + 8 + 1 + 7 = 18$$

$$a \to b \to d \to c \to a \qquad 2 + 3 + 1 + 5 = 11$$

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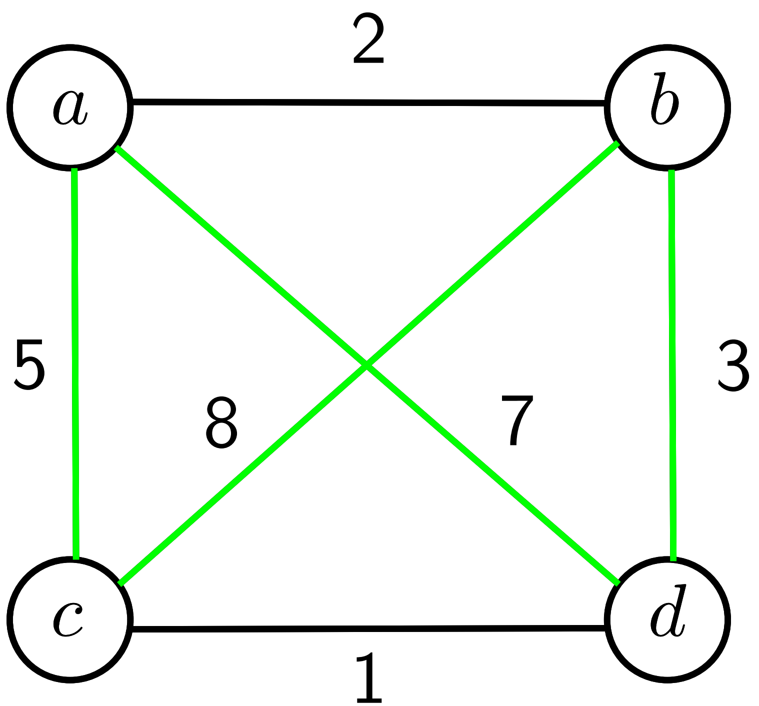

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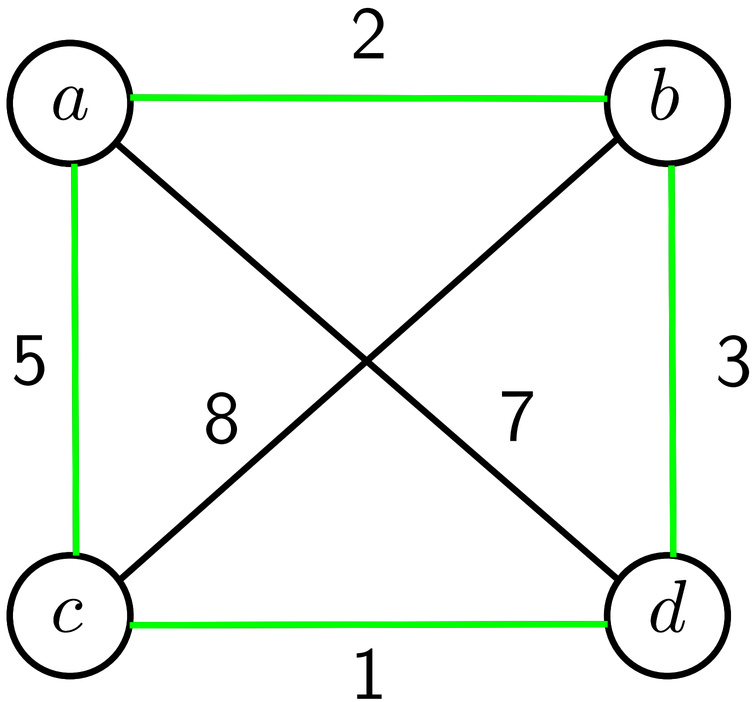

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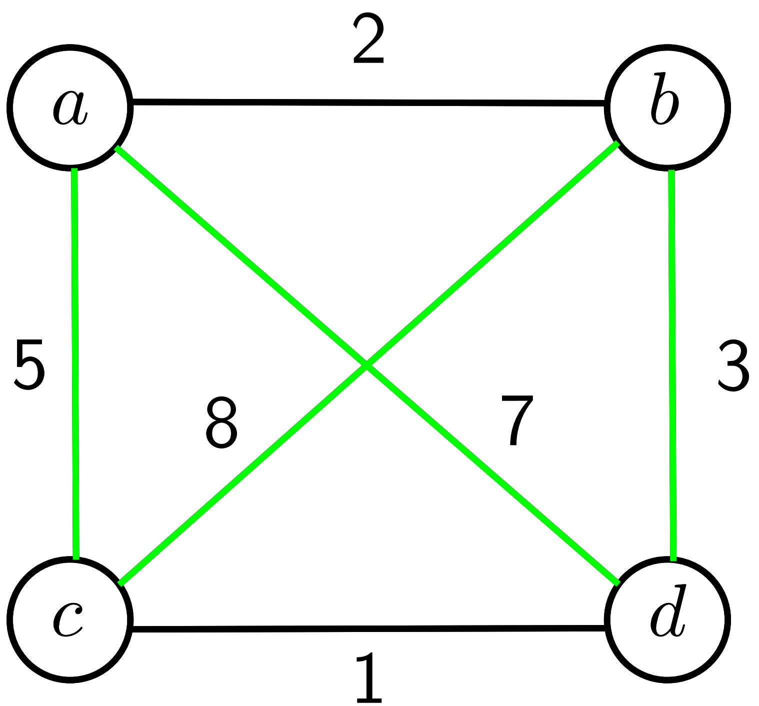

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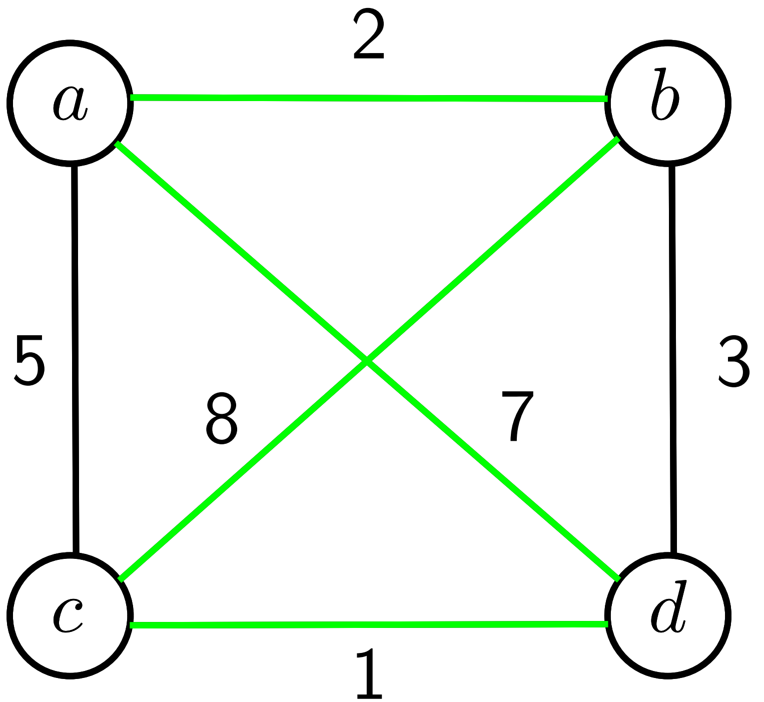

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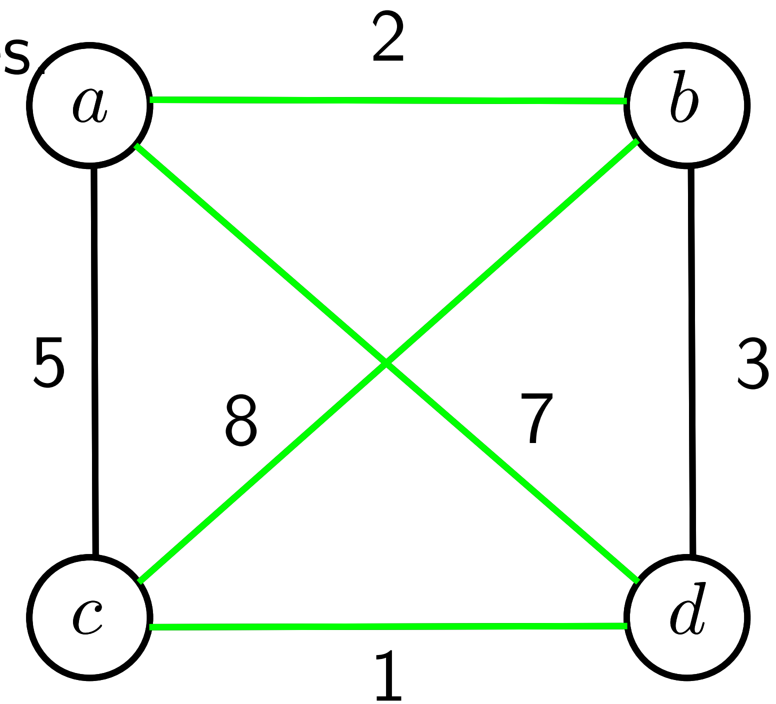

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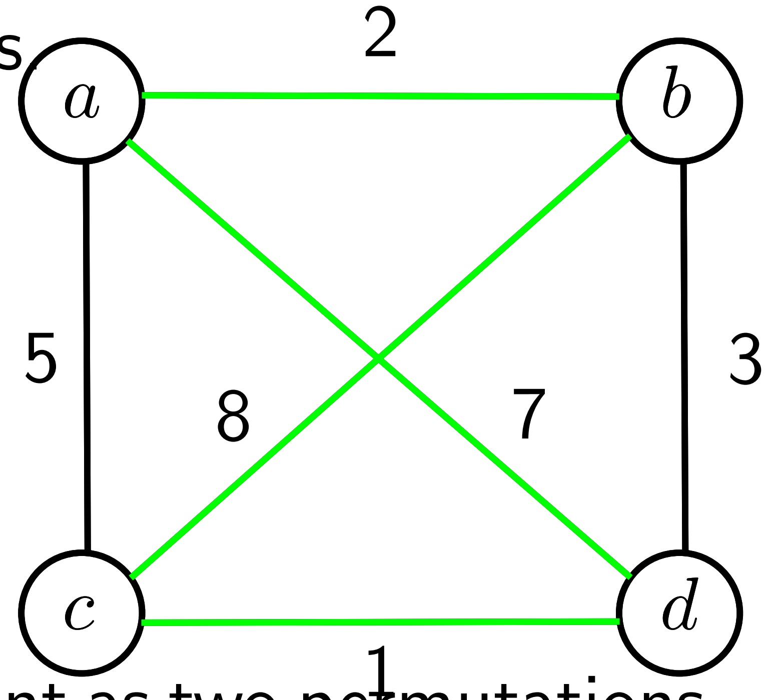

- Each permutation and its reverse count as two permutations.
- $\frac{1}{2}(n-1)!$ permutations of $n-1$ cities.
  But running time $= O(n!)$

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- If the problem size of for finding the closest pair is n, what is the size of the state space?
- What is the running time of brute force algorithm for finding the convex hull?

Questions!

- State the characteristics of brute force strategy.
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- What is Hamiltonian circuit?
- Generally brute force algorithms are not practical. Why?

Thank you.