SSN COLLEGE OF ENGINEERING
B.E. (Computer Science and Engineering) Semester 4
Unit Test: 1 (23 January 2017)

Time: 8.00–9.30 **CS6402 Design and Analysis of Algorithms** Max marks: 50

Part A $10 \times 2 = 20$

Answer *all* questions.

1. What is an algorithm? (CO1)
2. State the characteristic of *basic operations*. Which of the following are *not* basic operations? add, multiply, power, logical or. (CO1)
3. Find the order of growth of the function $10n^2 + 4n + 2$ with suitable values for c and n_0. (CO3)
4. If $f(x) = \frac{x^3}{2}$ and $g(x) = 37x^2 + 120x + 17$, show that $g = O(f)$, but $f \neq O(g)$. (CO3)
5. Find the order of growth of the sum $\sum_1^n (i^2 + 1)^2$ (CO3)
6. How many times the body of the inner loop is executed? What is the order of growth of the algorithm? (CO3)

```
for i ← 1 to m
    for j ← 1 to n
        c[i, j] ← a[i, j] + b[i, j]
    end
end
```

7. Find the time complexity of sum(a) where a is a list. (CO3)

Algorithm: sum $a[0 : n-1]$
if `a = []` **then return** 0
return a[0] + sum a[1:n-1]

8. Solve the recurrence relation: (CO3)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + 1 & \text{if } n > 1 \end{cases}$$

9. Prove that any comparison sort algorithm requires $\Omega(n \log n)$ comparisons in the worst case. (CO1)
10. For each of the following functions, indicate how much the functions value will change if its argument is increased fourfold. (CO3)
a. $\log_2 n$ b. $\sqrt{b}$ c. n d. n^2 e. n^3 f. 2^n

Part B $6 \times 5 = 30$

Answer any *five* questions.

11. Consider two algorithms A and B for solving the same problem running on two machines 1 and 2. Machine 1 executes 10^9 (1 billion) instructions per second, and machine 2 executes 10^7 (10 million) instructions per second. Algorithm A requires $2n^2$ instructions and runs on machine 1; algorithm B requires $50n \log_{10} n$ instructions and runs on machine 2. (CO1)
    (a) Calculate the running time of the two algorithms for inputs of sizes 100, 1000, 10000. (3)
    (b) Which is better — algorithm A on machine 1, or algorithm B on machine 2? Why? (3)

12. (a) Design a brute-force algorithm for finding the two closest points in a set of n points (the closest-pair problem). (CO2, 3)
    (b) Analyze the running time of the algorithm. (CO3, 3)

13. Given two $n \times n$ matrices A and B, write an algorithm for computing their product $C = AB$, and find its time efficiency. (CO2, CO3) (6)

14. (a) Design an algorithm to merge two sorted lists. Analyze its running time. (CO2, CO3, 3)
    (b) Design an algorithm to sort a list, dividing it into two almost equal sublists, sorting each sublist recursively, and then merging the two sorted sublists. Analyze the running time of this sort algorithm. (CO2, CO3, 3)

15. Derive a recurrence relation for Fibonacci series algorithm; also, carry out the time complexity. (CO3,6)

16. Analyze the best-case, the worst-case, and the average-case running times of the linear search algorithm for an array a of size n? (CO3, 2+2+2)

17. (a) Solve the recurrence relation (CO3, 3)

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

    (b) What is the order of growth of $T(n)$? (CO3, 3)

18. Design an algorithm to find all the common elements in two sorted lists of numbers. For example, for the lists 2, 5, 5, 5 and 2, 2, 3, 5, 5, 7, the output should be 2, 5, 5. What is the maximum number of comparisons your algorithm makes if the lengths of the two given lists are m and n, respectively? (CO2, CO3, 3+3)

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