Faculty Development Program

Design and Analysis of Algorithms

Introduction to Algorithms

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Contents

1. Outcomes

At the end of the course, the student will be able to:

- design algorithms for various computing problems,
- analyze the time and space complexity of algorithms,
- critically analyze the different algorithm design techniques for a given problem.
- modify existing algorithms to improve efficiency.

Books

- Anany Levitin, "Introduction to the Design and Analysis of Algorithms", Third Edition, Pearson Education, 2012.
- S. Dasgupta, C. H. Papadimitriou, and U. V. Vazirani, "Algorithms", McGraw-Hill Higher Education, 2006

2. Algorithmic Problem Solving

3. Important Problem Types

4. What is an Algorithm?

Algorithm

A well-defined computational procedure composed of of instructions that solves the problem in a finite number of steps.

- A computational problem is specified by its input, output, and the relation between the input and the output.
- Algorithms are procedural solution to problems.

Problem, Process, Algorithm

- A computational probelm is defined by the input given, the output required and the input–output relation (precondition and the postcondition).
- A computational process starts with the precondition, evolves, and terminates with the postcondition.
- A computational process is generated by an algorithm. When an algorithm is executed, a process evolves.

5. Design, correctness, efficiency

- Specify the problem precisely: its input, output, and input–output relation.
- Design an algorithm for solving a problem.
- Prove the correctness of the algorithm: when the algorithm is executed, the specified input–output relation is satisfied
- Analyze the efficiency of the algorithm: the resources (running time and memory space) required by the algorithm to finish execution.

6. Problem specification

> Speed takes you nowhere if you are heading in the wrong direction.
>
> American proverb

- Understand the computational process (or invent a process).
- Execute the process with examples.
- Specify the computational problem in terms of its input, output, and the relation between the input and the output.
- Is there an algorithm in the manual?
- If there area many, compare algorithms.

7. Computing constraints

- Computing model: von Neumann machine, RAM random access machine $\rightsquigarrow$ sequential algorithms
- Resources available vs huge volumes of data and time-criticality
- Exact vs approximate algorithms

8. Design algorithms and data structures

- Visualize the process.
- Conceptualize the algorithm.
- Data representation is the essence of programming.
- Algorithms + Data structures = Programs

Algorithmic notation

- Natural language (English) — too ambiguous but informal.
- Programming language (even a high-level one) — too tedious but precise.
- Pseudocode: A blend of English and high-level programming language — informal and precise.
- Very-high level programming languages — Python, Haskell
- How about flowchart (flawchart)?

Algorithm	Program
Need not be formal (but unambiguous and precise)	Formal (hence, unambiguous and precise)
For design, correctness, analysis	Can be executed by a machine
Must terminate	Need not terminate

9. Design techniques

- Algorithm design techniques or strategies or tools — general approaches to common to a variety of problems
- Classification

10. Proving correctness

> Correctness is clearly the prime quality. If a system does not do what it is supposed to do, then everything else about it matters little.
>
> — Bertrand Meyer

> Program testing can be a very effective way to show the presence of bugs, but is hopelessly inadequate for showing their absence.
>
> — E W Dijkstra

- A correct program need still be tested.
- Need to be simple to understand.

There are theories to reason about the correctness of programs.

Criticism:

- Most of the programs are not mission-critical!

- Learning curve of the theory is steep, and the benefit not commensurate with the effort.
- Real programs too large to calculate! So why calculate “small” programs?

Benefit:

- The goal is not correctness for correctness’ sake.
- The goal of program construction is to satisfy the input-ouput relation. Concern for correctness leads the stepwise construction of program toward the goal. The end program is correct by construction.
- Scale of the artifact does not obviate the need for knowing the fundamental tools of the trade!

Loop invariant and progress measure

Mathematical induction on iterative algorithms

- Linear search: search for a target in an array.
- Polish National Flag: partition an array into two parts.

Search for a target in an array, Linear search

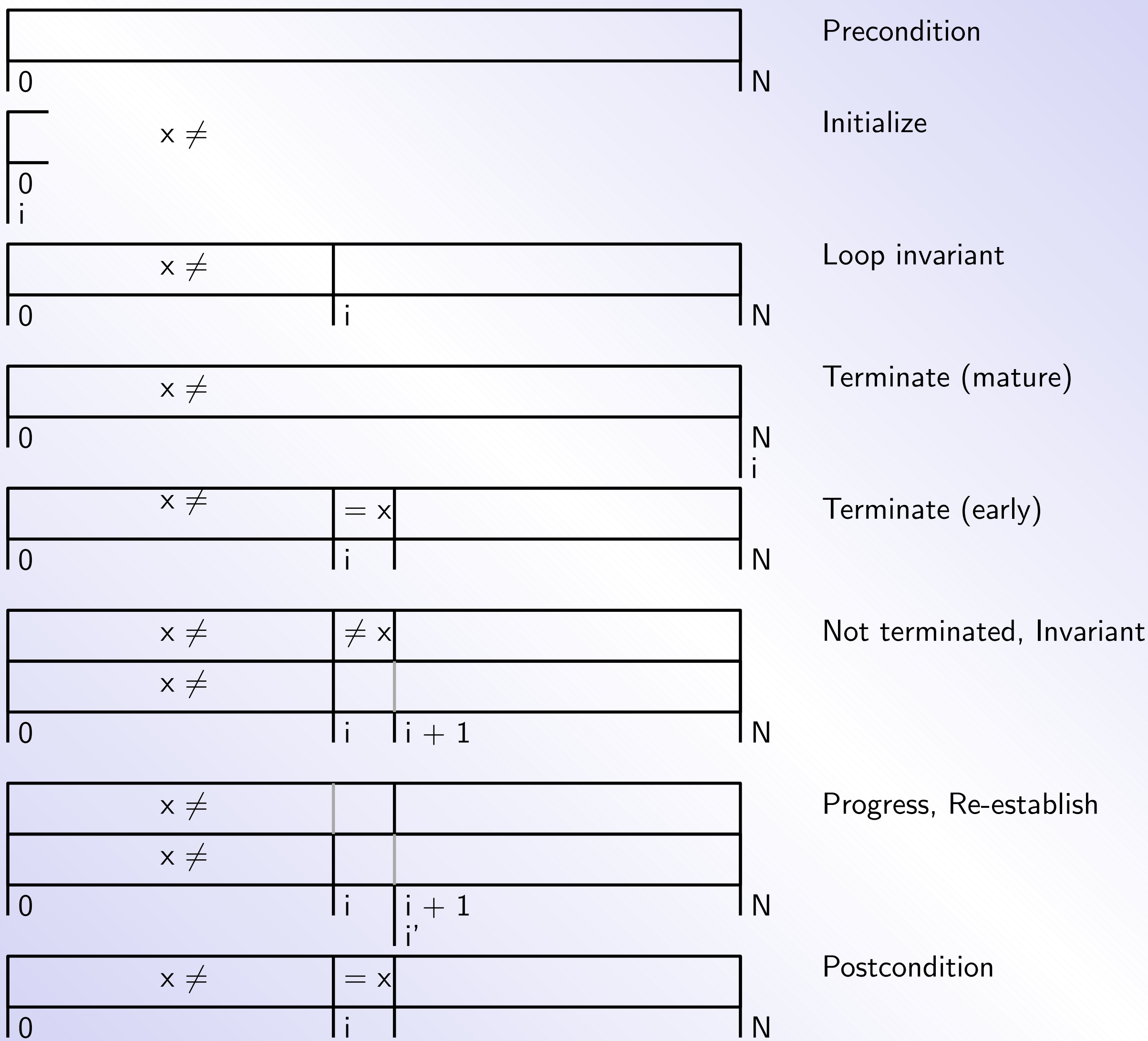
The input are an array `a` of N comparable items and a target item `x` to be searched in the array. The postcondition is: the output is the index of the target item in the array, if the target is present in the array; otherwise, the output is `N`, index of an invalid item.

A snapshot in the middle of the loop is that the subarray `a[0:i]` is processed (or conversely subarray `a[i:N]` is unprocessed). The basic iterative step is to compare the next item `a[i]` with the target `x`. In an iterative step, the processed subarray `a[0:i]` grows (unprocessed subarray `a[i:N]` shrinks).

Input: `a = [2, 5, 6, 8, 9, 10], N = 6, x = 9`

iteration	i	a	x
1	0	2, 5, 6, 8, 9, 10	9
2	1	2, 5, 6, 8, 9, 10	9
3	2	2, 5, 6, 8, 9, 10	9
4	3	2, 5, 6, 8, 9, 10	9
5	4	2, 5, 6, 8, 9, 10	9

Output: `i = 4`



Precondition
0
N
Initialize
x ≠
0
i
Loop invariant
x ≠
0
i
N
Terminate (mature)
x ≠
0
N
i
Terminate (early)
x ≠
= x
0
i
N
Not terminated, Invariant
x ≠
≠ x
x ≠
0
i
i + 1
N
Progress, Re-establish
x ≠
x ≠
0
i
i + 1
i'
N
Postcondition
x ≠
= x
0
i
N

Algorithm: LinearSearch (a, x)

input : An array a = [a_0, a_1, ..., a_{N-1}] of size N

output: Index i such that x $\neq$ [0:i] and a[i] = x or i = N

```
i ← 0
until i = N or a[i] = x do
    i ← i + 1
end
return i
```

Algorithm: LinearSearch (a, x)

input : An array a = [a_0, a_1, ..., a_{N-1}] of size N

output: Index i such that x ≠ [0:i] and a[i] = x or i = N

```
i ← 0
-- i = 0, x ∉ [0:i] ≡ x ∉ []
until i = N or a[i] = x do
    -- i ≠ N, x ∉ [0:i], x ≠ [i]
    -- x ∉ [0:i+1]
    i ← i + 1
    -- x ∉ [0:i']
end
-- x ∉ [0:i], x = [i]
-- x ∉ [0:i], i = N ≡ x ∉ [0:N]
return i
```

Partition an array into two parts, Polish National Flag

The input is an array a of `N` comparable items. The items are of two kinds, say Red and White. The postcondition is that the array is partitioned into two subarrays and the items so rearranged that

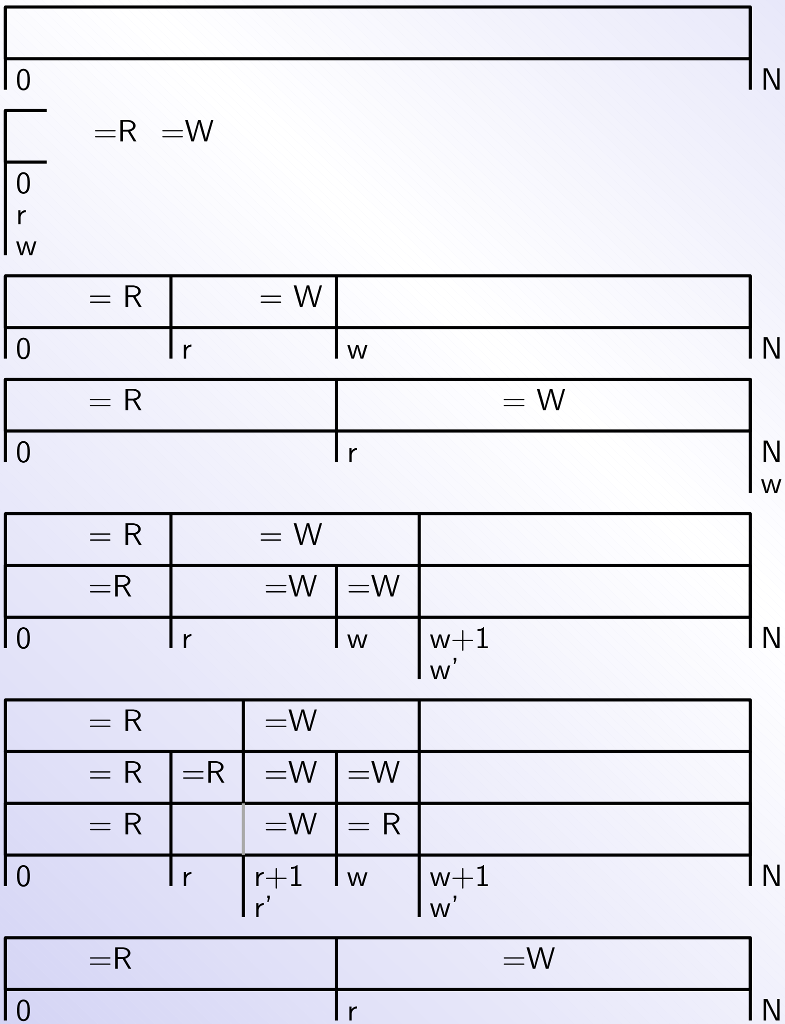
- subarray `[0:r]` has items of one kind (red), and
- subarray `[r:N]` has items of the other kind (white)

The only operations permitted are

- compare two items
- swap two items

Computer Science is no more about computers than astronomy is about telescopes or biology is about microscopes (E W Dijkstra).

Dutch National Flag



Precondition `r, w = 0, 0`

Initialize

Loop invariant

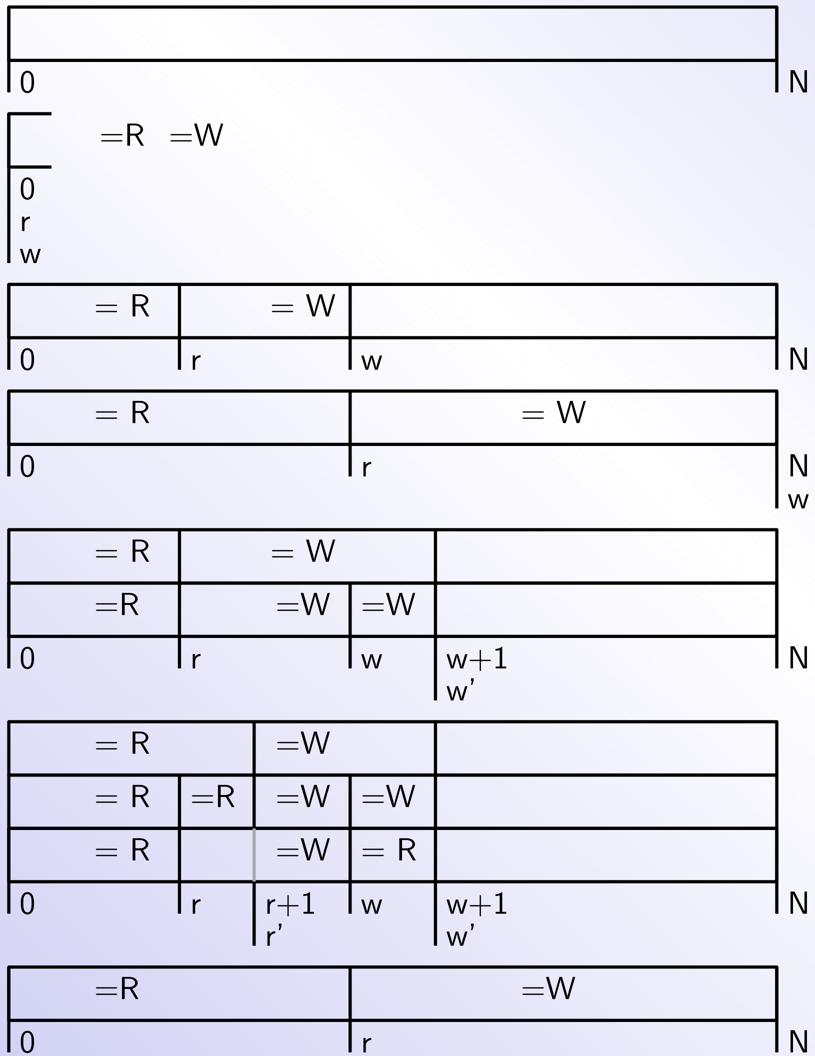
Terminate `w == N`

Progress, Re-establish `[w] == W | w' = w+1`

Progress, Re-establish

```
[w] == R |
    swap [r] [w]
    r, w = r+1, w+1
```

Postcondition



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Progress, Re-establish

```
[w] == R |
    swap [r] [w]
    r, w = r+1, w+1
```

Postcondition

Algorithm: Partition (a)

input : A array a[0:N] of red and white items

output: Index `r` such that a[0:r] is red, a[r:N] is white

```
r, w ← 0, 0
until w = N do
    if a[w] = W then
        w ← w+1
    else
        swap (A, r, w)
        r, w ← r+1, w+1
    end
end
return r
```

Algorithm: Partition (a)

input : A array a[0:N] of red and white items

output: Index r such that a[0:r] is red, a[r:N] is white

```
r, w ← 0, 0
-- [] is red, [] is white
until w = N do
    -- [0:r] is red, [r:w] is white
    if a[w] = W then
        w ← w+1
        -- [0:r] is red, [r:w'] is white
    else
        swap (A, r, w)
        r, w ← r+1, w+1
        -- [0:r'] is red, [r':w'] is white
    end
end
return r
```

11. Analyzing algorithms for efficiency

- Time efficiency — how fast
- Space efficiency — how much memory

12. Syllabus

Unit I 9

Algorithm Analysis: Time Space Trade-off – Asymptotic Notations – Conditional asymptotic notation – Removing condition from the conditional asymptotic notation – Properties of big-Oh notation – Recurrence equations – Solving recurrence equations – Analysis of linear search.

Unit II 9

Divide and Conquer: General Method – Binary Search – Finding Maximum and Minimum – Merge Sort. Greedy Algorithms: General Method – Container Loading Knapsack Problem.

Unit III 9

Dynamic Programming: General Method – Multistage Graphs – All-Pair shortest paths – Optimal binary search trees – 0/1 Knapsack – Traveling salesperson problem.

Unit IV 9

Backtracking: General Method – 8 Queens problem – sum of subsets – graph coloring – Hamiltonian problem – knapsack problem.

Unit V 9

Graph Traversals: Connected Components – Spanning Trees – Biconnected components. Branch and Bound: General Methods (FIFO & LC) – 0/1 Knapsack problem. Introduction to NP-Hard and NP-Completeness.

Text Books

- Ellis Horowitz, Sartaj Sahni and Sanguthevar Rajasekaran, "Computer Algorithms/C++", Second Edition, Universities Press, 2007. (For Units II to V)
- K.S. Easwarakumar, "Object Oriented Data Structures using C++", Vikas Publishing House pvt. Ltd., 2000 (For Unit I)

References

- T. H. Cormen, C. E. Leiserson, R.L.Rivest, and C. Stein, "Introduction to Algorithms", Second Edition, Prentice Hall of India Pvt. Ltd, 2003.
- Alfred V. Aho, John E. Hopcroft and Jeffrey D. Ullman, "The Design and Analysis of Computer Algorithms", Pearson Education, 1999.