Traveling Salesman Problem

there are only 2^n different subsets of n objects $(n! > 2^n)$. Let $G = (V, E)$ be a directed graph with edge costs c_{ij}. The variable c_{ij} is defined such that $c_{ij} > 0$ for all i and j and $c_{ij} = \infty$ if $\langle i, j \rangle \notin E$. Let $|V| = n$ and assume $n > 1$. A *tour* of G is a directed simple cycle that includes every vertex in V. The cost of a tour is the sum of the cost of the edges on the tour. The *traveling salesperson problem* is to find a tour of minimum cost.

$$g(1, V - \{1\}) = \min_{2 \leq k \leq n} \{c_{1k} + g(k, V - \{1, k\})\} \tag{5.20}$$

Generalizing (5.20), we obtain (for $i \notin S$)

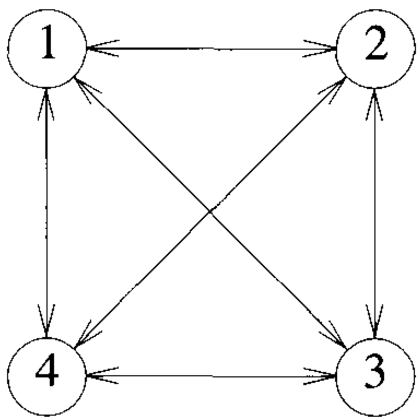
$$g(i, S) = \min_{j \in S} \{c_{ij} + g(j, S - \{j\})\} \tag{5.21}$$

$$g(i, \phi) = c_{i1}, \ 1 \leq i \leq n.$$

Contd…

of size 1. Then we can obtain $g(i, S)$ for S with $|S| = 2$, and so on. When $|S| < n - 1$, the values of i and S for which $g(i, S)$ is needed are such that $i \neq 1$, $1 \notin S$, and $i \notin S$.

Example



(a)

0	10	15	20
5	0	9	10
6	13	0	12
8	8	9	0

(b)

Thus $g(2, \phi) = c_{21} = 5, g(3, \phi) = c_{31} = 6$, and $g(4, \phi) = c_{41} = 8$. Using (5.21), we obtain

$$
\begin{array}{lllll}
g(2, \{3\}) & = & c_{23} + g(3, \phi) & = & 15 \\
g(3, \{2\}) & = & 18 & & \\
g(4, \{2\}) & = & 13 & &
\end{array}
\qquad
\begin{array}{lll}
g(2, \{4\}) & = & 18 \\
g(3, \{4\}) & = & 20 \\
g(4, \{3\}) & = & 15
\end{array}
$$

Next, we compute $g(i, S)$ with $|S| = 2$, $i \neq 1$, $1 \notin S$ and $i \notin S$.

$$\begin{aligned} g(2,\{3,4\}) &= \min\ \{c_{23} + g(3,\{4\}), c_{24} + g(4,\{3\})\} &= 25 \\ g(3,\{2,4\}) &= \min\ \{c_{32} + g(2,\{4\}), c_{34} + g(4,\{2\})\} &= 25 \\ g(4,\{2,3\}) &= \min\ \{c_{42} + g(2,\{3\}), c_{43} + g(3,\{2\})\} &= 23 \end{aligned}$$

Finally, from (5.20) we obtain

$$\begin{aligned} g(1,\{2,3,4\}) &= \min\{c_{12} + g(2,\{3,4\}), c_{13} + g(3,\{2,4\}), c_{14} + g(4,\{2,3\})\} \\ &= \min\{35, 40, 43\} \\ &= 35 \end{aligned}$$

Contd…

j that minimizes the right-hand side of (5.21). Let $J(i, S)$ be this value. Then, $J(1, \{2, 3, 4\}) = 2$. Thus the tour starts from 1 and goes to 2. The remaining tour can be obtained from $g(2, \{3, 4\})$. So $J(2, \{3, 4\}) = 4$. Thus the next edge is $\langle 2, 4 \rangle$. The remaining tour is for $g(4, \{3\})$. So $J(4, \{3\}) = 3$. The optimal tour is 1, 2, 4, 3, 1. $\square$