Algorithm Analysis

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1. For each of the following algorithms, indicate (i) a natural size metric for its inputs, (ii) its basic operation, and (iii) whether the basic operation count can be different for inputs of the same size:

   a. computing the sum of n numbers

   b. computing $n!$

   c. finding the largest element in a list of n numbers

   d. Euclid's algorithm

   e. sieve of Eratosthenes

   f. pen-and-pencil algorithm for multiplying two n-digit decimal integers

a. (i) n; (ii) addition of two numbers; (iii) no

b. (i) the magnitude of n, i.e., the number of bits in its binary representation; (ii) multiplication of two integers; (iii) no

c. (i) n; (ii) comparison of two numbers; (iii) no (for the standard list scanning algorithm)

d. (i) either the magnitude of the larger of two input numbers, or the magnitude of the smaller of two input numbers, or the sum of the magnitudes of two input numbers; (ii) modulo division; (iii) yes

e. (i) the magnitude of n, i.e., the number of bits in its binary representation; (ii) elimination of a number from the list of remaining candidates to be prime; (iii) no

f. (i) n; (ii) multiplication of two digits; (iii) no

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	3		5		7		9		11		13		15		17		19		21		23		25
2	**3**		5		7				11		13				17		19				23		25
2	3		**5**		7				11		13				17		19				23		

ALGORITHM *Sieve*(n)

//Implements the sieve of Eratosthenes
//Input: A positive integer $n > 1$
//Output: Array L of all prime numbers less than or equal to n

for $p \leftarrow 2$ **to** n **do** $A[p] \leftarrow p$
for $p \leftarrow 2$ **to** $\lfloor \sqrt{n} \rfloor$ **do** //see note before pseudocode
    if $A[p] \neq 0$ //p hasn't been eliminated on previous passes
        $j \leftarrow p * p$
        while $j \leq n$ **do**
            $A[j] \leftarrow 0$ //mark element as eliminated
            $j \leftarrow j + p$
//copy the remaining elements of A to array L of the primes
$i \leftarrow 0$
for $p \leftarrow 2$ **to** n **do**
    if $A[p] \neq 0$
        $L[i] \leftarrow A[p]$
        $i \leftarrow i + 1$
return L

a. Can the problem of computing the number π be solved exactly?

b. How many instances does this problem have?

Which of the following formulas can be considered an algorithm for computing the area of a triangle whose side lengths are given positive numbers a, b, and c?

a. $S = \sqrt{p(p-a)(p-b)(p-c)}$, where $p = (a+b+c)/2$

b. $S = \frac{1}{2}bc \sin A$, where A is the angle between sides b and c

c. $S = \frac{1}{2}ah_a$, where h_a is the height to base a

Old World puzzle A peasant finds himself on a riverbank with a wolf, a goat, and a head of cabbage. He needs to transport all three to the other side of the river in his boat. However, the boat has room for only the peasant himself and one other item (either the wolf, the goat, or the cabbage). In his absence, the wolf would eat the goat, and the goat would eat the cabbage. Solve this problem for the peasant or prove it has no solution. (Note: The peasant is a vegetarian but does not like cabbage and hence can eat neither the goat nor the cabbage to help him solve the problem. And it goes without saying that the wolf is a protected species.)