TSP

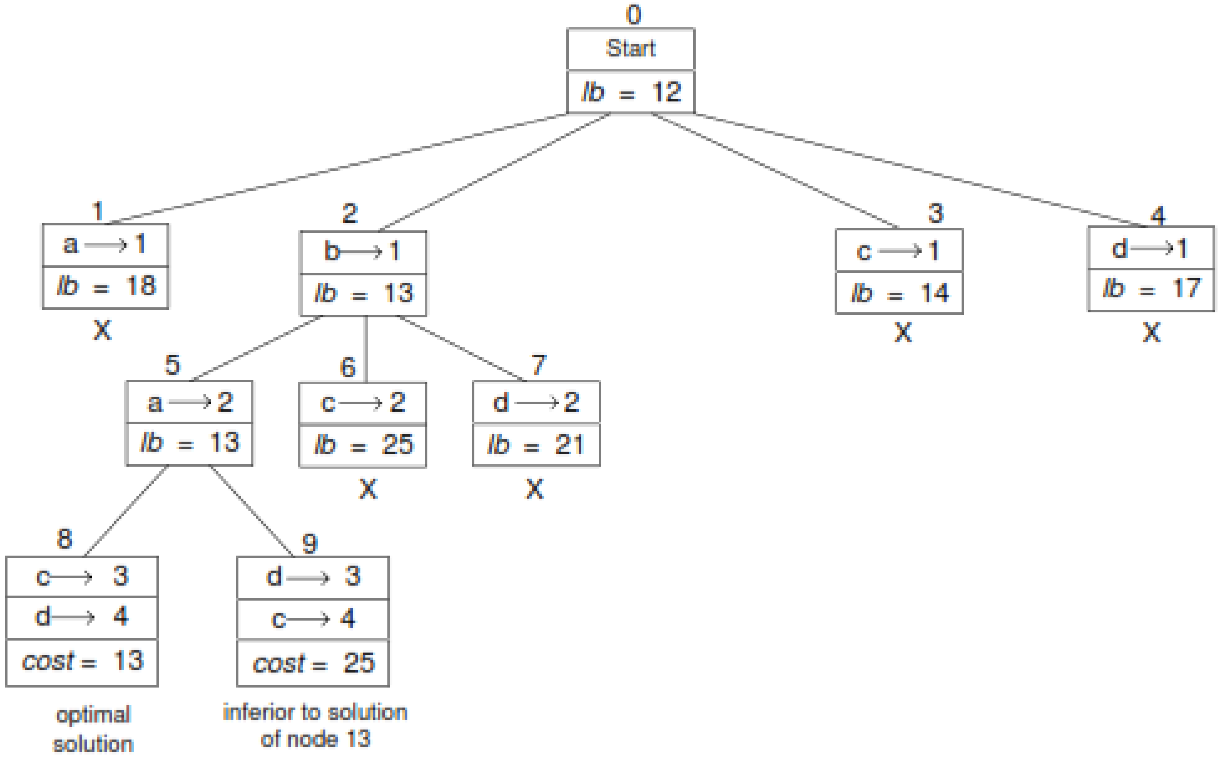
Assignment Problem

- Solve the same instance of the assignment problem as the one solved in the section by the best-first branch-and-bound algorithm with the bounding function based on matrix columns rather than rows.

2. The instance discussed in the section is specified by the matrix

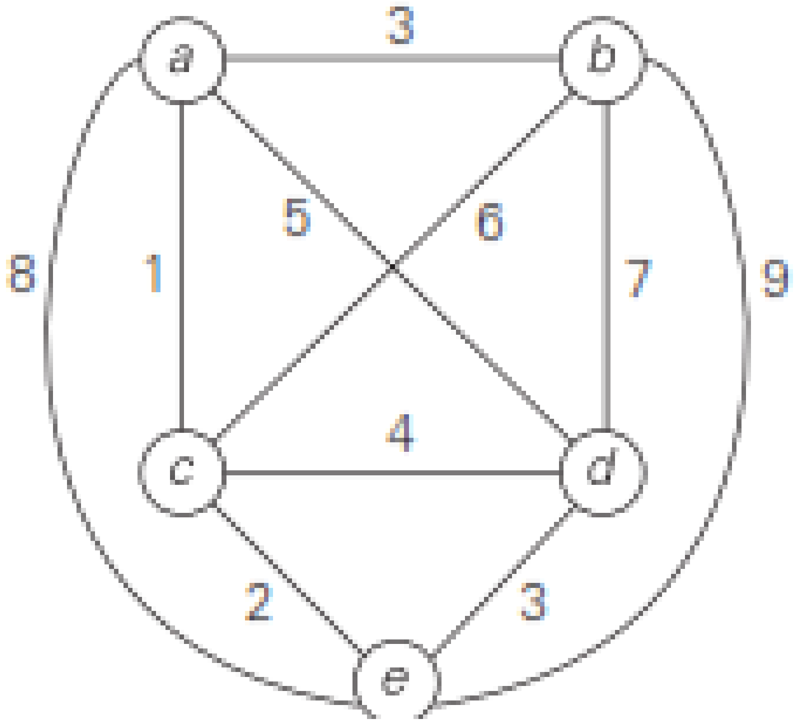
$$C = \begin{array}{cccc} \text{job 1} & \text{job 2} & \text{job 3} & \text{job 4} \\ 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{array} \begin{array}{l} \\ \text{person } a \\ \text{person } b \\ \text{person } c \\ \text{person } d \end{array}$$

Here is the state-space tree in question:



The optimal assignment is $b \to 1$, $a \to 2$, $c \to 3$, $d \to 4$.

TSP



5 .

$$lb = \lceil s/2 \rceil. \tag{12.2}$$

For example, for the instance in Figure 12.9a, formula (12.2) yields

$$lb = \lceil [(1+3) + (3+6) + (1+2) + (3+4) + (2+3)]/2 \rceil = 14.$$

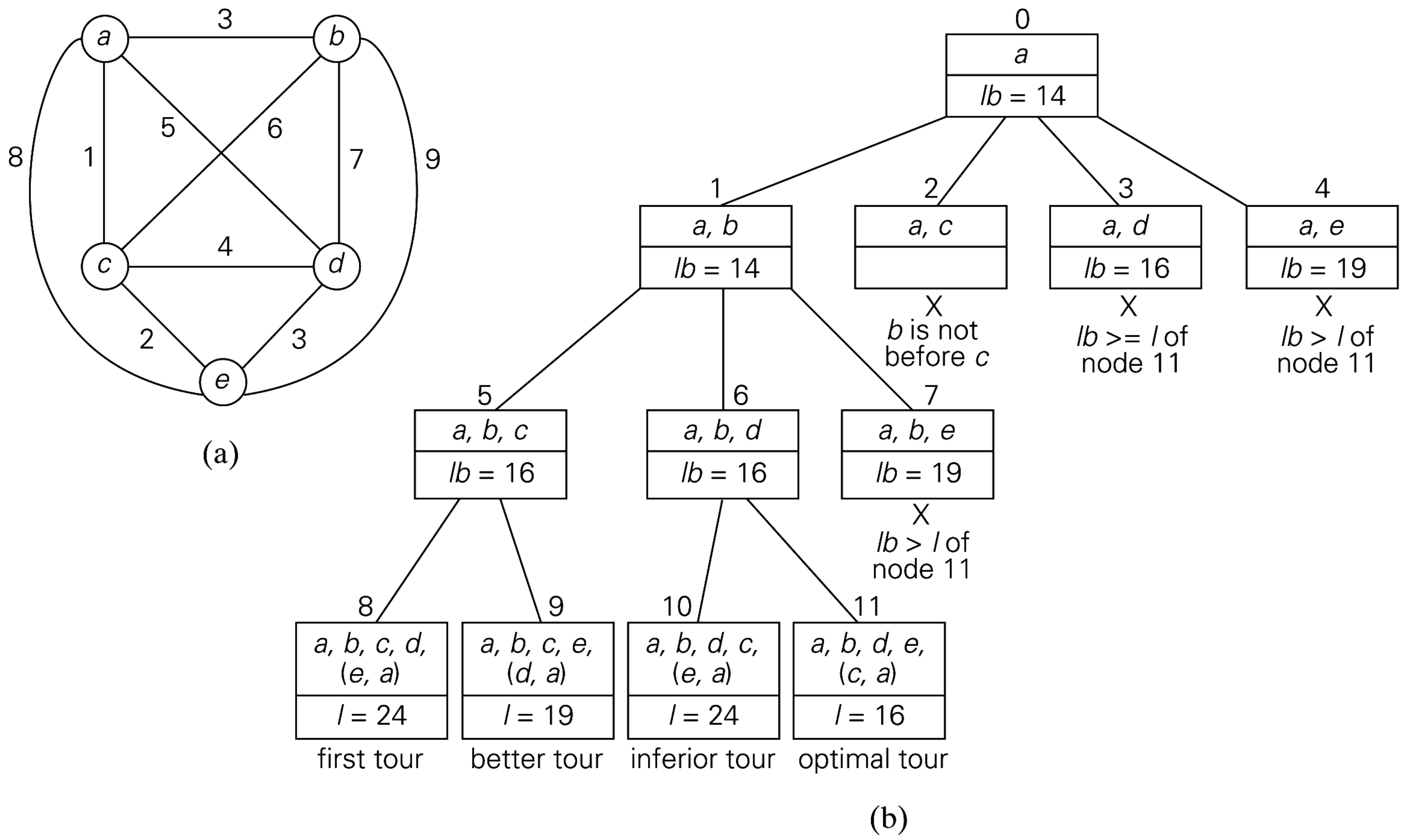
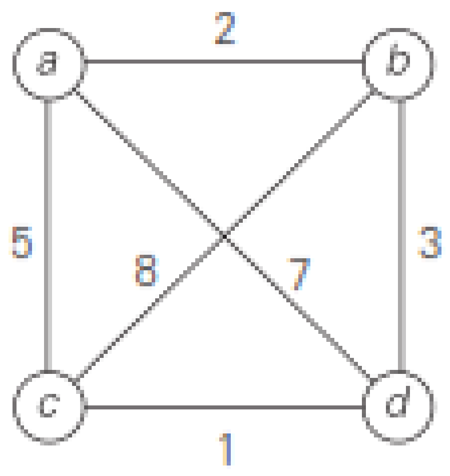
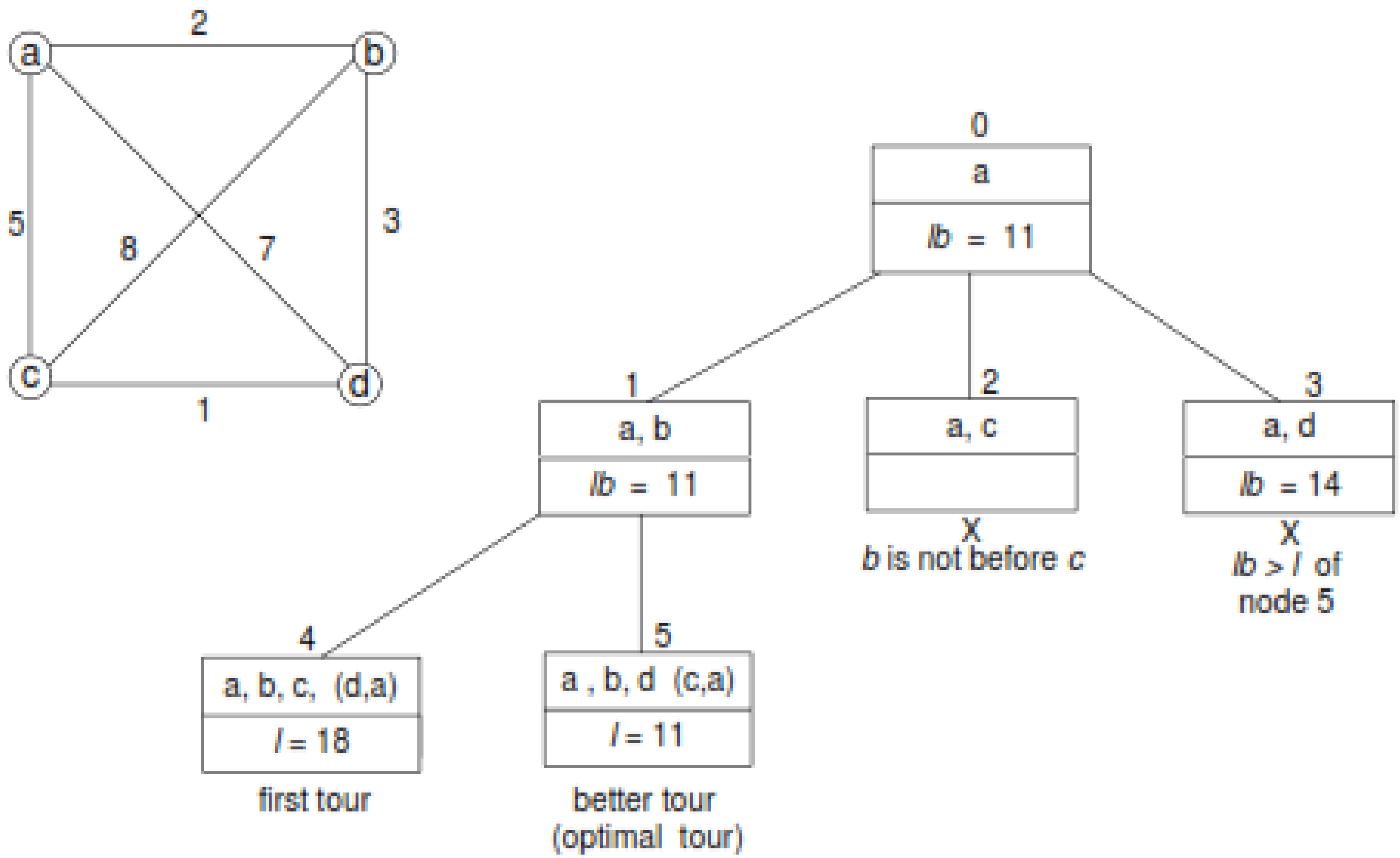


FIGURE 12.9 (a) Weighted graph. (b) State-space tree of the the branch-and-bound algorithm to find the shortest Hamiltonian circuit in this graph. The list of vertices in a node specifies a beginning part of the Hamiltonian circuits represented by the node.

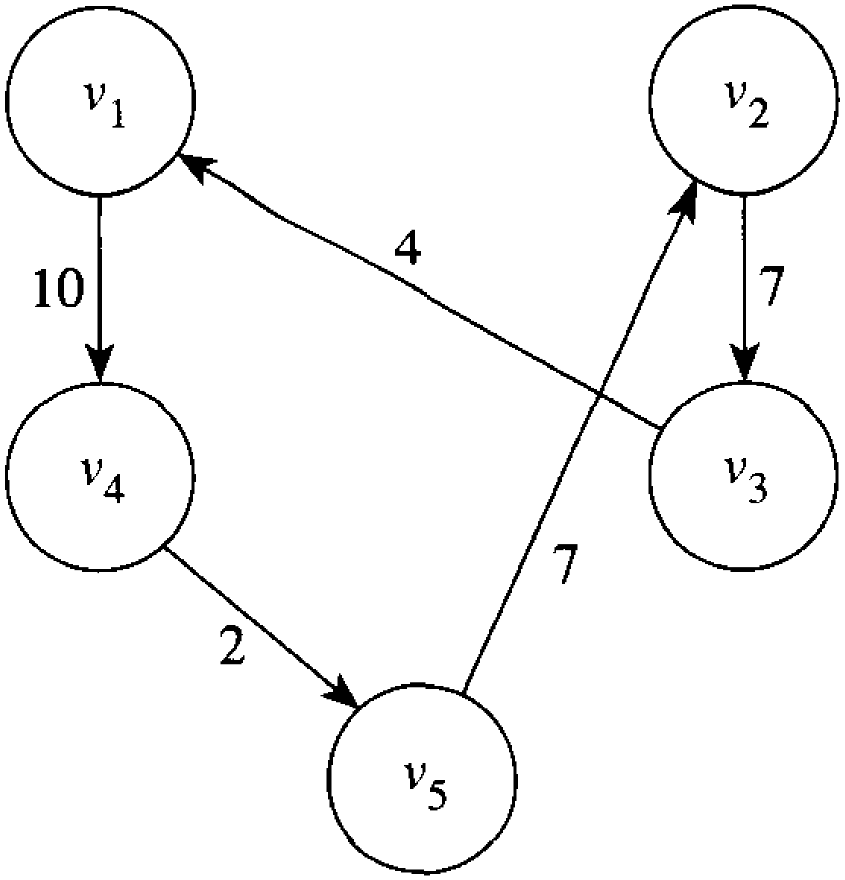


2
a
b
5
8
7
3
c
d
1

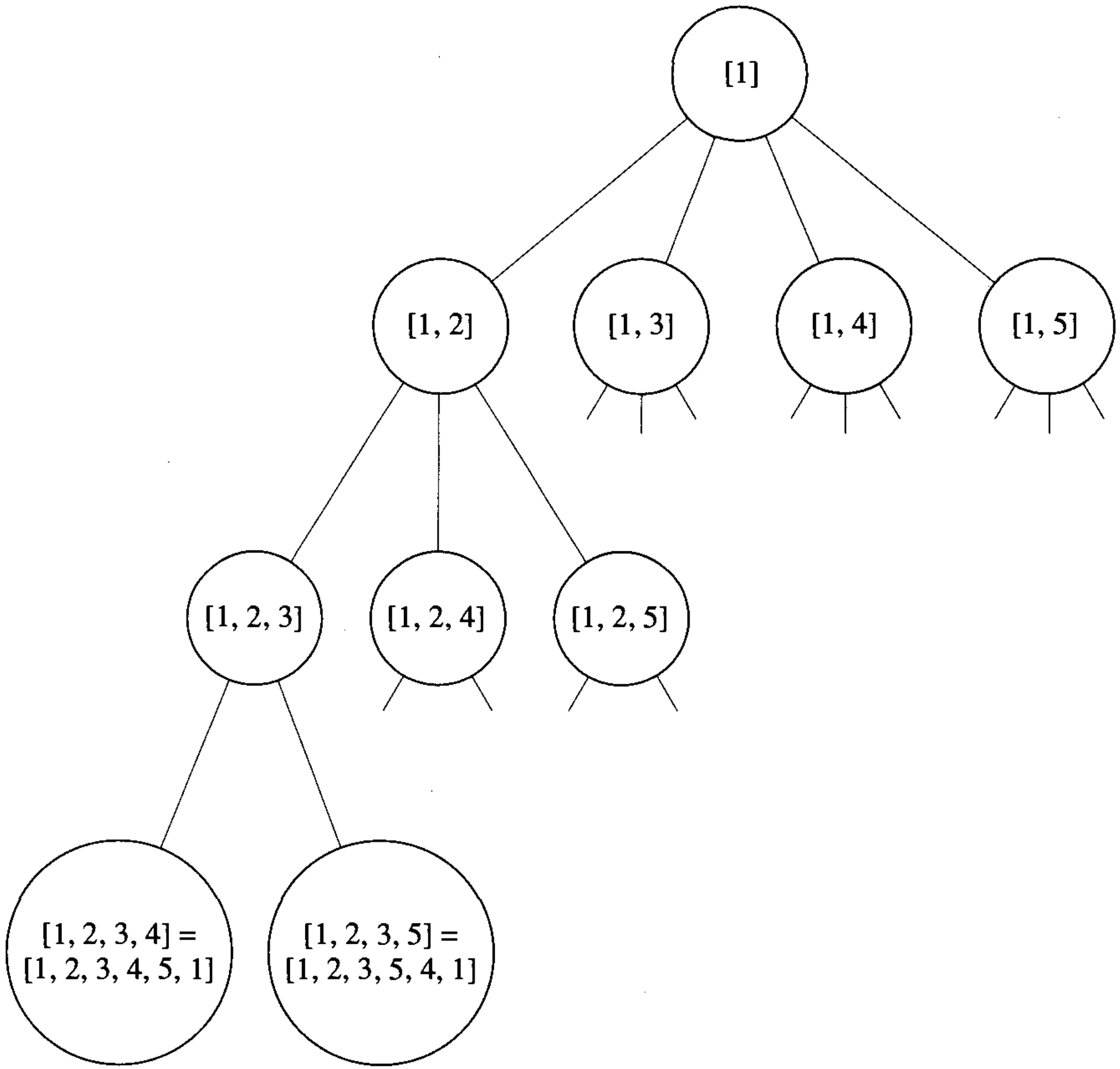


The found optimal tour is a, b, d, c, a of length 11.

$$\begin{bmatrix} 0 & 14 & 4 & 10 & 20 \\ 14 & 0 & 7 & 8 & 7 \\ 4 & 5 & 0 & 7 & 16 \\ 11 & 7 & 9 & 0 & 2 \\ 18 & 7 & 17 & 4 & 0 \end{bmatrix}$$



v_1	$minimum(14, 4, 10, 20)$	$= 4$
v_2	$minimum(14, 7, 8, 7)$	$= 7$
v_3	$minimum(4, 5, 7, 16)$	$= 4$
v_4	$minimum(11, 7, 9, 2)$	$= 2$
v_5	$minimum(18, 7, 17, 4)$	$= 4$


[1]
[1, 2]
[1, 3]
[1, 4]
[1, 5]
[1, 2, 3]
[1, 2, 4]
[1, 2, 5]
[1, 2, 3, 4] =
[1, 2, 3, 4, 5, 1]
[1, 2, 3, 5] =
[1, 2, 3, 5, 4, 1]

Lower bound

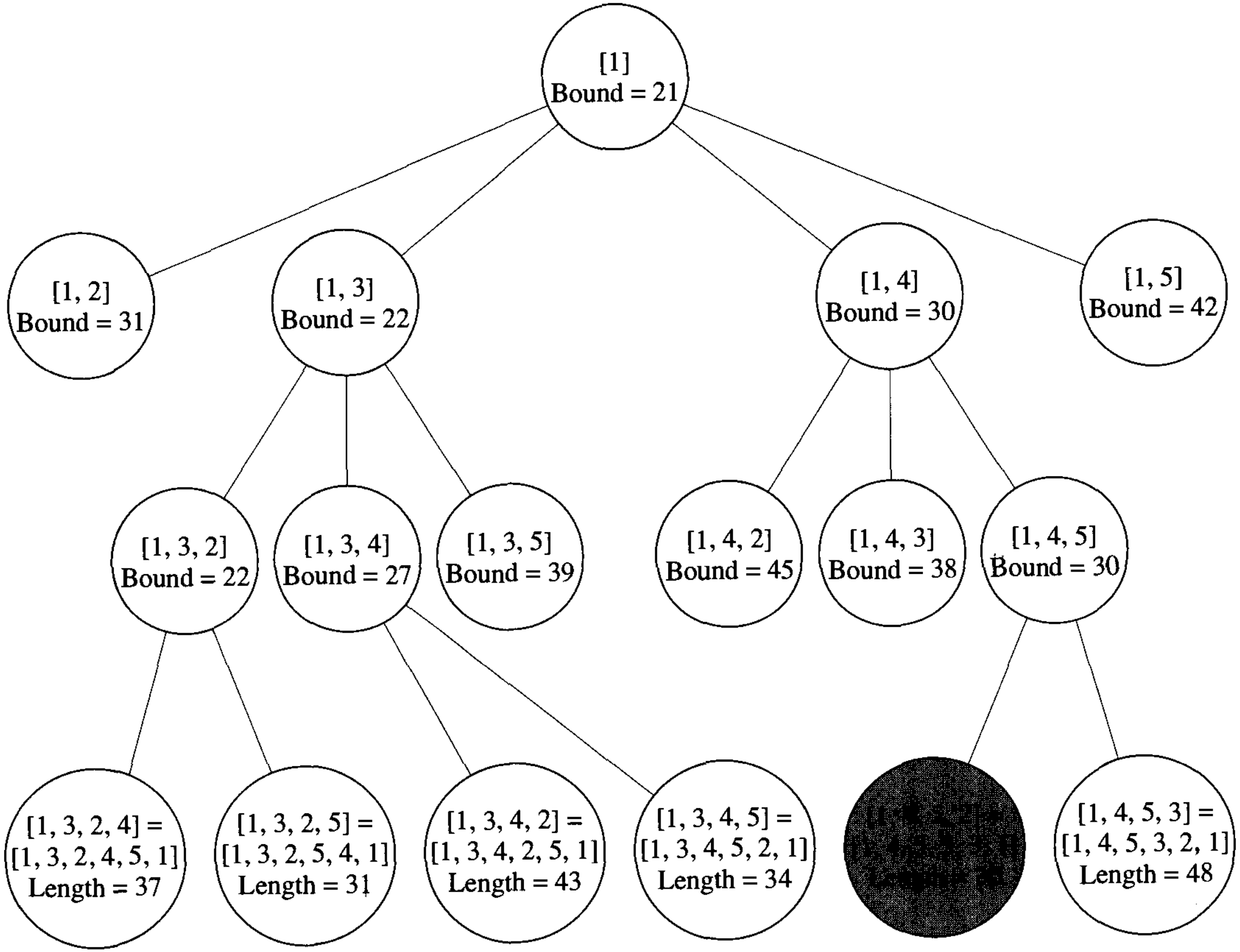
$$4 + 7 + 4 + 2 + 4 = 21.$$

Node(1,2)

$$
\begin{array}{lll}
v_1 & & 14 \\
v_2 & minimum(7, 8, 7) & = 7 \\
v_3 & minimum(4, 7, 16) & = 4 \\
v_4 & minimum(11, 9, 2) & = 2 \\
v_5 & minimum(18, 17, 4) & = 4
\end{array}
$$

Node(1,2,3)

v_1		14
v_2		7
v_3	$minimum(7, 16) =$	7
v_4	$minimum(11, 2) =$	2
v_5	$minimum(18, 4) =$	4


[1]
Bound = 21
[1, 2]
Bound = 31
[1, 3]
Bound = 22
[1, 4]
Bound = 30
[1, 5]
Bound = 42
[1, 3, 2]
Bound = 22
[1, 3, 4]
Bound = 27
[1, 3, 5]
Bound = 39
[1, 4, 2]
Bound = 45
[1, 4, 3]
Bound = 38
[1, 4, 5]
Bound = 30
[1, 3, 2, 4] =
[1, 3, 2, 4, 5, 1]
Length = 37
[1, 3, 2, 5] =
[1, 3, 2, 5, 4, 1]
Length = 31
[1, 3, 4, 2] =
[1, 3, 4, 2, 5, 1]
Length = 43
[1, 3, 4, 5] =
[1, 3, 4, 5, 2, 1]
Length = 34
[1, 4, 5, 3] =
[1, 4, 5, 3, 2, 1]
Length = 48